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„Horizontal and vertical product differentiation combined –

Integrating quality, horizontal differentiation and expansion, and price setting into a multidimensional Hotelling problem“

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PREFACE

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1. INTRODUCTION

1.1 Abstract

This paper will attempt to create a theoretical model integrating horizontal and vertical product differentiation as well as optimal price setting.

Horizontal differentiation will occur in two different ways: First — analogously to the standard HOTELLING[1] model — by choosing a **location** along a spectrum of individual customer preferences (product or characteristic space[2]), and second, by choosing a length of **horizontal expansion** extending symmetrically from the chosen location in its centre, allowing the producer to design a product simultaneously and equally covering more than one individual preference at a time.

Vertical differentiation will occur by choosing a level of produced quality equally perceived by all potential customers, regardless of the horizontal location of their individually preferred characteristic. Only one single level of quality is chosen, resulting in quasi-fixed cost independent of the number of units produced.

Optimal price setting will occur by determining one single optimal price valid for every customer regardless of their horizontal location of their preferred characteristic (*mill pricing*).

[1] HOTELLING (1929), pp. 41-57

[2] CARLTON & PERLOFF (2005), p. 206

I.e., a firm may *not* differentiate prices, setting different prices for different market segments/ consumers.

First, a **monopolist**'s optimal choice of the strategic variables location, horizontal expansion, quality, and price will be determined, assuming that there is **no threat of entry**.

Second, we will examine an incumbent's possible **entry deterrence** strategies in *oligopolistic competition with free entry*.[3] On the part of the entrant, three different **strategies of entry** will be discussed: *Central entry and competition*, *Central entry and peripheral competition*, and *Peripheral entry and competition*. These must in turn be anticipated by the incumbent, whose respective deterrence strategies will be compared with regard to their **preemptive** effect.

1.2 Limitation

Please note that in this context, no incumbent strategies other than deterrence, like *accommodation* (letting the entrant profitably take location, thus sharing the market), will be discussed.

[3] At this point, oligopolistic competition with free entry must be distinguished from monopolistic competition. Both face downward-sloping demand curves and the total annihilation of profit, but in monopolistic competition, a price change by one firm has only negligible effect on the demand of any other firm (e.g. TIROLE (1994), p. 247). As we will see in this model as well, this is not so in oligopolistic competition.

For a discussion of monopolistic competition, see Edward Hastings CHAMBERLIN (1933), e.g. in CHAMBERLIN (1965): "The Theory of Monopolistic Competition: A Re-orientation of the Theory of Value", Cambridge, Harvard University Press, 8th ed., or, for an overview, e.g. CARLTON & PERLOFF (2005), ch. 7 (pp. 200ff).

For a discussion on the *extent* of entry, see SALOP (1979) (also reviewed in TIROLE (1994), ch. 7.1.2 (pp. 282ff), as well as in CARLTON & PERLOFF (2005), pp. 223ff)

2. MODEL ELEMENTS

First we will introduce the model's strategic variables (in chapter 2.1), second, the exogenous parameters (chapter 2.2). Subsequently follow compound terms that will be of great interest (2.3), and, on the basis of all of those, the construction of the model itself (in 2.4).

2.1 Strategic variables

As stated, the model developed throughout chapter 2 will incorporate as strategic variables the monopolist's or incumbent's horizontal location, horizontal expansion, (vertical) quality, and price.

The incumbent must set all four strategic variables before the potential entrant sets his (sequential game; this will be of importance in chapter 4 *Behaviour under Threat of Entry*, and i.e. in 4.2 *Strategies of Entry and Entry Deterrence*). In this, the model at hand differs from the classical models of HOTELLING 1929 and SALOP 1979, where location is either exogenously given or where all players simultaneously take up location *first* before any one sets his price in a second phase.

A firm's choice of strategic variables is a binding commitment, and may therefore not be altered later in the game.

All strategic variables are non-negative by nature.

2.1.1 Location

As in the standard models (HOTELLING 1929, SALOP 1979), a "location" is chosen among possible values of a one-dimensional characteristic of finite length (product or characteristic space[4]). "Location" can e.g. be interpreted as

- **Geographic location:** one geographic location (e.g. of an outlet, or a service-providing device like a broadcasting tower or mobile telecommunications mast) amidst the (actual or preferred) locations of customers along one dimension (e.g. a street),

- **Variant:** one variant of a product among other variants that are favoured by other customers (equally many, given uniform distribution of customer preferences along the dimension of "location"),

- **Design:** a design among other possible designs of equal quality,

- **Sensory characteristic:** An example for a limited but continuous scale where it is plausible that individual preferences are equally distributed is the **colour** spectrum of visible light. A main colour may be chosen from the spectrum — or any other sensory characteristic that may be viewed being a choice from a more or less continuous spectrum of perceivable (and potentially enjoyable) possibilities (like e.g. beats per minute).

- **Complexity:** a versatile but complex technology or user interface versus an easy-to-use interface with limited but sufficient functionality. *Early adopters*[5] like high-end technology users and beta testers usually prefer the first, while *laggards* or the *late majority* may prefer the latter. Or, connoisseurs of visual arts, music or avantgarde cuisine may prefer more complex oeuvres or artful concepts, compositions or arrangements, or more experimental tastes, while others may prefer "products" that are easier to follow or to enjoy.

[4] CARLTON & PERLOFF (2005), p. 206

[5] terminology employed by Everett M. ROGERS in: ROGERS , Everett M., (1962), "Diffusion of Innovations", Free Press of Glencoe, Macmillan Company

Location is therefore a *horizontal* criterion, as it reflects *individual* preference of one variant to others, rather than an objective/intersubjective characteristic where *every* customer would, cet. par., prefer one value to another.

In the model at hand, every location is available at the same cost (zero).

Every player must choose one single location (no brand proliferation by a single player). And as stated above, every player has to choose his location *simultaneously* with his other strategic variables. Location choice is *sequential* only in the sense that the incumbent chooses his location (as well as all his other strategic variables) before the entrant sets his location (and the rest of his strategic variables). As stated above, in this the game differs from the standard models of HOTELLING 1929 and SALOP 1979, where all players take up location first before any one sets his price in a second phase. Sequential location choice in a linear product space has been discussed by PRESCOTT & VISSCHER (1977)[6], SCHMALENSEE (1978)[7], JUDD (1982), and Lambertini (2002)[8].

2.1.2 Horizontal expansion λ

Whereas earlier models (as HOTELLING 1929 or SALOP 1979[9]) only allowed for pinpoint locations, in the model at hand the producer may choose to offer a *length* of horizontal "expansion" extending symmetrically from the chosen location in its centre, allowing the producer to design a product simultaneously covering more than one individual preference at a time. Thus, consumers located within horizontal expansion λ do not incur transportation costs.

[6] Edward C. PRESCOTT & Michael VISSCHER (1977): "Sequential location among firms with foresight", The Bell Journal of Economics, Vol. 8, No. 2 (Autumn 1977), pp. 378-393. The full text is publicly accessible at the RAND Corporation website http://www.jstor.org/discover/10.2307/3003293

[7] Richard SCHMALENSEE (1978): "Entry Deterrence in the Ready-to-Eat Breakfast Cereal Industry", The Bell Journal of Economics, Vol. 9, No. 2 (Autumn 1978), pp. 305-327

[8] Luca LAMBERTINI (2002): "Equilibrium locations in a spatial model with sequential entry in real time", Regional Science and Urban Economics, Elsevier, Vol. 32, No. 1 (January), pp. 47-58

[9] a discussion of Salop's model can e.g. be found in TIROLE (1994), ch. 7.1.2 (pp. 282ff), as well as in CARLTON & PERLOFF (2005), pp. 223ff

The model at hand assumes progressively (quadratically) increasing **horizontal expansion costs**:

$$K_\lambda(\lambda) = k_\lambda \cdot \lambda^2 \qquad [2.1]$$

Interpretations of horizontal expansion may include:

- **Opening hours:** Costs increase progressively with the offered time interval as wages per hour rise, e.g. when standard working time elapses and supplemental hours begin. One problem with this interpretation may be the assumption of uniformly distributed consumers' preferred time for shopping. Another simplification differing from reality that this interpretation requires would be a shopping time duration of zero, as the individual choice from the offered expansion is one preferred value only, rather than an interval.

- **Geographic distribution range:** If location designates the (sole) region in which the producer decides to settle, geographic distribution range may designate the range within which a dense distribution network is established. Progressive costs may arise e.g. from ever-more-costly logistics.

- **Performance range:** Progressively costly technology allowing for flawless performance within the chosen range (e.g. telecommunications broadcasting). Beyond performance range, broadcasting quality or bandwidth begins to decrease until no bandwidth is left (see (linear) transportation costs).

- **Customization** of the chosen basic variant (corresponding to the chosen "location"): Customizing elements may become progressively more expensive as the customized variants deviate more and more from the basic product. This model assumes a one-dimensional continuum of variants (e.g. quantum supercomputer—mainframe system—large or business network—small or private network—tabletop workstation—notebook—mini note-book – palmtop, or convertible—sportscar—coupé—limousine—van—SUV—real offroader, assuming every feature or technical part could be customized to suit either the former or the next variant, thus creating a quasi continuum of possible in-between-variants); it would have to be modified in order to suit products that are horizontally differentiated in multiple dimensions.

- **Feature sets:** The basic product ("location") may come along in more or less variants including more (e.g. for the high-end users) or less (e.g. for those preferring an easy-to-use interface) features or applications.

- **Sensory sets**, e.g. colour sets: The more colours or other sensory characteristics that individual consumers might value have to be integrated into one single product without thwarting individual "aesthetic utility", the more elaborated the design will have to be.

Horizontal expansion thus constitutes a variable that serves a length of adjoining locations of consumer preferences.

2.1.3 Quality *q*

By way of quality, vertical product differentiation is integrated in the model. Quality is **identical to utility** gained from the product, and is *equally* perceived and valued by *all* consumers, regardless of the location of their individually preferred variant.

$$u_q(q) \equiv q \qquad [2.2]$$

As the respective cost function employs a cost factor k_q , no additional factor translating quality into utility is needed.[10] Therefore, the willingness to pay for quality is linear as well.

An example for less intuitive interpretations of quality may be that of

[10] This can be shown as follows:

Assume a factor translating quality into utility, *consumer sensitivity to quality* s_q :

$$u_q(q) = s_q \cdot q$$

Then the quadratic cost function to produce this level of consumer utility would be:

$$K_q(u) = \kappa_q \cdot u_q^2 = \kappa_q \cdot s_q^2 \cdot q^2$$

But as both κ_q and s_q^2 are *constant*, we can *aggregate* those two factors to a single factor, $k_q := \kappa_q \cdot s_q^2$, yielding:

$$K_q(q) = k_q \cdot q^2$$

Therefore, only *one* factor (k_q) is needed to relate produced consumer utility to cost.

- **Transmission power:** Beyond the short broadcasting range (within which broadcasting quality does not vary / decrease), transmission power may determine total broadcasting range. With linear "transportation costs", broadcasting quality would linearly decrease with increasing distance from the flawless broadcasting range.

The model at hand assumes progressively (quadratically) increasing **quality production costs** (see 2.3.4).

Only one quality level of quality may be chosen and offered to all customers.

Quality directly benefits **net consumer surplus** (see 2.4.4).

2.1.4 Price *p*

In the model at hand, every firm must set *one* single price for all potential customers (a firm may *not* differentiate price for different market segments/customers). As stated above, price setting has to be effected *simultaneously* with the choice of the firm's other strategic variables location, horizontal expansion, and quality.

Analogously to quality, **consumer disutility** equals price; no additional disutility function is employed. Thus, disutility from price is **linear**, and normalised to match the units of price (factor equal to 1):

$$u_p(p) = -p \qquad [2.3]$$

Price also equals **gross revenue per purchasing consumer** — every customer buys *one* unit of the product if his net surplus is non-negative (see transportation costs). Gross revenue per purchasing consumer is decreased by marginal cost (per unit produced) to equal gross margin (see marginal cost).

Price directly affects **net consumer surplus** (see 2.4.4).

2.2 Exogenous parameters

Contrary to the above strategic variables, the following parameters are not subject to the monopolist's choice and consequently cannot be optimized, but are exogenous, constant and, in oligopolistic competition with free entry (chapter 4), equal for all players.

All parameter values are non-negative.

2.2.1 Number of consumers *N*

N designates the total number of consumers present along the spectrum of preferences *L* (see below), and equally, as every consumer purchases either one or zero products, the maximum number of sales (sold units) possible.

2.2.2 Length of characteristic space *L*

N consumers are, in this model, **uniformly distributed** over characteristic space, that is, the length of the spectrum of preferred variants, *L*.

A parameter derived from the number of consumers *N* and length *L*, which is, as we will see, of interest for the optimization calculation, is **consumer density *N/L*** (see 2.3.1).

2.2.3 Transportation cost factor τ

τ is the factor by which the distance between the location of an offered product and the locations of consumers' preferred variants is decreasing their utility, and thus probably the most influential parameter in this model. In other terms, τ is the **factor** by which **distance** discounts **utility**, thus resulting in the *net* surplus of a consumer whose preferred variant is located at a certain distance from an offered variant.

Correspondingly, $1/\tau$ is the factor by which a certain level of consumer surplus can be translated into a **range of attraction** to any side, within which consumers will still benefit from a non-negative *net* surplus despite the remoteness of their preferred variant from the location where said *maximum* consumer surplus is being offered (see also section 2.4.6 *Range of attraction* r_j).

2.2.4 Marginal costs *c*

Marginal costs designate the costs of production for one unit of the product that is subsequently sold to a customer demanding it.

In the model at hand, marginal costs are **constant**ly equal to *c* — rather than decreasing with the number of units produced, which would reflect classical economies of scale. Nevertheless, the model does show economies of scale, as unit production costs are independent of quality, which in turn causes quantity-independent costs, which again can be "spread" on more (economies of scale) or less units to cover quality production costs. Thus, given a certain quality level, quantity *does* reduce unit price, in the end.

2.2.5 Quality cost factor k_q

The factor by which squared quality q^2 causes costs (see 2.3.4 *Quality production costs*).

2.2.6 Horizontal expansion cost factor k_λ

The factor by which squared horizontal expansion λ^2 causes costs (see 2.3.5 *Horizontal expansion costs*).

2.2.7 Fixed and entry costs

In the calculation, **fixed costs** are omitted due to their trivial subtractive nature.

We do not assume **entry costs**, which would entail further discussion in terms of entry and entry deterrence. The known effects include that if entry costs are *sunk* costs, the incumbent, having already entered the market, does not incur entry costs, whereas the entrant would need to bear entry costs. For this reason, the incumbent would not need to relinquish the *totality* of his profits, but may, in a one-phase game, keep a profit up to the entrant's entry costs, and still deter him. See e.g. TIROLE 1994 (chapter 8) for a detailed discussion of these issues.

2.3 Crucial compound terms

The following compound terms are neither strategic variables, nor do they constitute *additional* exogenous parameters, but are derived from one or, usually, more-than-one of the former.

All values are non-negative.

2.3.1 Consumer density *N/L*

Consumer density *N/L*, the quotient of the number of consumers *N* over characteristic space *L*, is another (obviously non-independent) parameter which, as we will see, is of interest for profit calculation and, therefore, optimization.

2.3.2 Maximum consumer surplus $u_\lambda(p, q)$

Maximum consumer surplus designates the largest possible utility that can be generated by a consumer located *within horizontal expansion* λ (therefore incurring no transportation cost) by purchasing at price p one unit of a product of quality q. Maximum consumer surplus is equal to the difference between quality q and price p:

$$u_\lambda(p, q) = q - p \qquad [2.4]$$

Maximum consumer surplus will be a crucial item in oligopolistic competition with free entry (chapter 4).

Please note that we will repeatedly use the abbreviated form “consumer surplus” whenever it clearly follows from context that this means “maximum consumer surplus” (whereas we will never omit the “net” in “net consumer surplus”, see section 2.4.4).

2.3.3 Gross margin ($p - c$)

Gross margin, ($p - c$), or the difference of collected price minus marginal cost, is another element of the profit function, and therefore regularly shows in the optimization calculation.

2.3.4 Quality production costs $K_q(q)$

As stated in section 2.2.4 *Marginal costs*, a chosen quality level does not impact on *unit* production costs, but causes (part of total) **overhead costs** amounting to:

$$K_q(q) = k_q \cdot q^2 \qquad [2.5]$$

Thus, quality production costs, the exogenous quality production costs *factor* k_q aside (see 2.2 *Exogenous parameters*, section 2.2.5), solely depend on the **squared** level of quality q^2.

2.3.5 Horizontal expansion costs $K_\lambda(\lambda)$

The (part of total) overhead costs of offering a positive horizontal expansion (see 2.1 *Strategic variables*, section 2.1.2) are proportional to the *expansion cost factor* k_λ (see 2.2 *Exogenous parameters*, section 2.2.6) and increase **quadratically** with *horizontal expansion* λ :

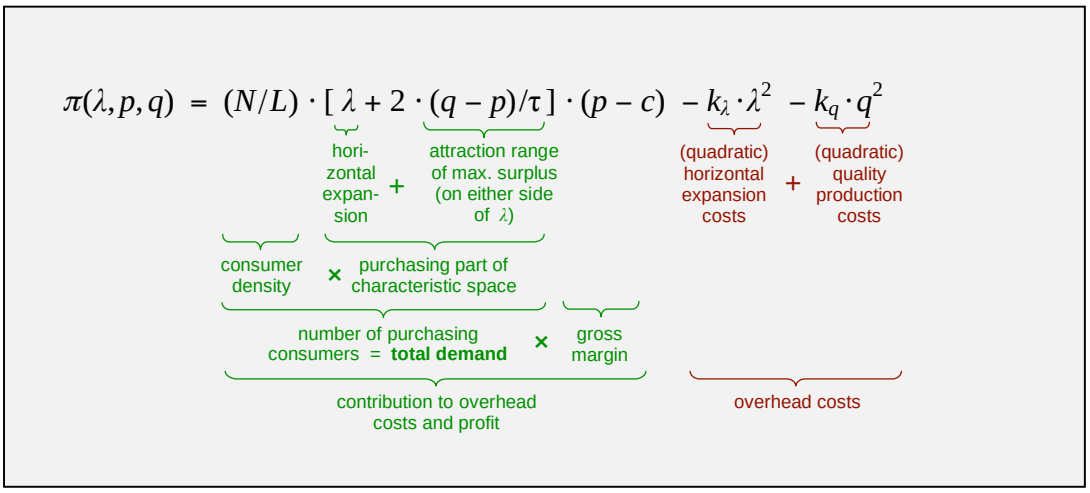
$$K_\lambda(\lambda) = k_\lambda \cdot \lambda^2 \qquad [2.6]$$

2.3.6 Profit π

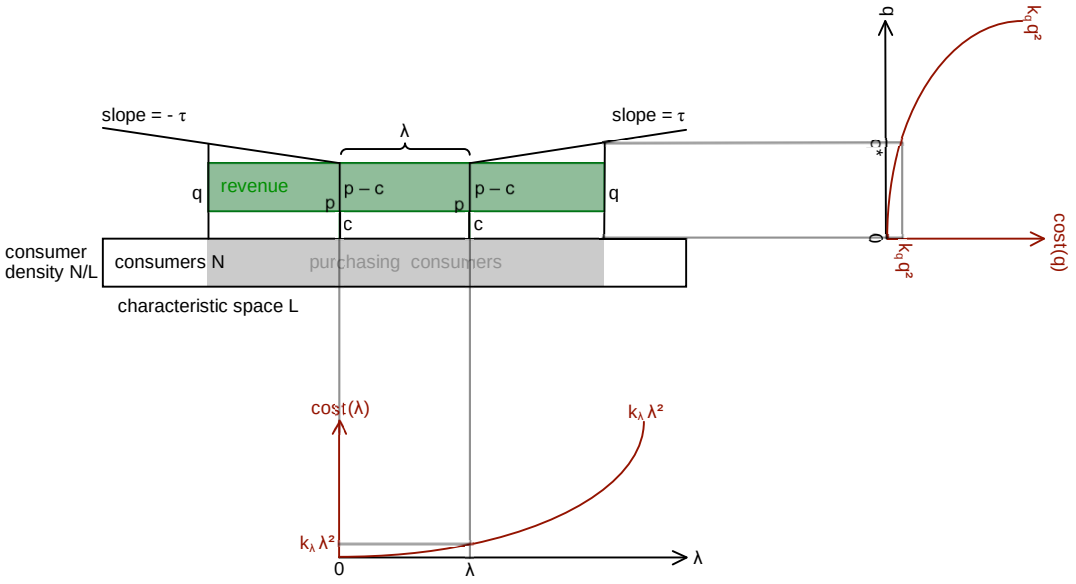
The producers' profits are their central interest, and thus the term to be maximized, respectively, be it the monopolist's profit, incumbent's, or the potential entrant's.

Ignoring all constraints as well as the problems in optimizing location and arising by competition, profit primarily depends on the choice of the strategic variables horizontal expansion λ, price p, and quality q, and, making use of the elements defined in the above sections 2.3.1 through 2.3.5, can be expressed as shown in Table 2.1:

Table 2.1: The profit function $\pi(\lambda, p, q)$ and its elements



The following graphics (Graphics 2.I) illustrate the interaction of the elements of the profit function as defined in Table 2.1:



Graphics 2.I: Interaction of the elements of the profit function $\pi(\lambda, p, q)$

2.4 Consumer behaviour

In general, consumers are assumed to strive for maximization of their individual utility (see sections 2.4.2 to 2.4.4), i.e. with respect to their individual location. Consumers are assumed to rationally decide *whether* or not to purchase in case of one single offer, and, in addition, *which* offer to purchase in case that more than one offer is capable of increasing individual utility.

Chapter 2.4 will succinctly discuss consumer endowment (section 2.4.1), consumer utility from quality (2.4.2), consumer disutility from price and transportation costs (2.4.3), net consumer surplus (2.4.4), individual demand (2.4.5), and, following from that, a producer's range of attraction (2.4.6) and total demand (2.4.7).

2.4.1 Consumer endowment

We assume that every consumer's income or endowment allows him to potentially purchase one unit of the product at equilibrium price.

2.4.2 Elements of consumer utility

Consumer utility is, for every consumer, **identical to quality**:

$$u_{qi}(q) \equiv q \qquad [2.7]$$

As was shown in 2.1.3, in view of the progressive costs of producing consumer utility, additional factors (e.g. *sensitivity to quality*) translating quality into utility are obsolete.[11]

From a given quality, all consumers obtain the *same* level of utility (i.e., there is no *distribution* of sensitivity to quality as e.g. in GABSZEWICZ & THISSE (1979) or SHAKED & SUTTON (1982).

2.4.3 Elements of consumer disutility

As with quality, consumer disutility from price is assumed to be identical for every consumer and equal to p:

$$u_{pi}(p) = -p \qquad [2.8]$$

Disutility from individual **transportation costs** is assumed to be linear and equal to transportation costs as well:

$$u_{\tau i}(z_i^*) = -\tau \cdot |z_i - z_i^*| \qquad [2.9]$$

where τ is the *transportation costs factor* (see section 2.2.3), z_i consumer *i*'s location in horizontal characteristic space *L*, and z_i^* the *nearest location* offered by a producer[12]. Consequently, if z_i lies *within* a producer's horizontal expansion λ, then the consumer does not incur any transportation costs with this producer at all.

2.4.4 Net consumer surplus u_i

Following from [2.7] through [2.9], consumer *i*'s net surplus resulting from the overall transaction in question equals

$$u_i(p, q, z_i^*) = q - p - \tau \cdot |z_i - z_i^*| \qquad [2.10]$$

2.4.5 Individual demand

Same as the standard location models (HOTELLING 1929, SALOP 1979) this model assumes unit demand, i.e., every consumer *i* can and will purchase either zero or one unit (at the most).

---

[11] This can be shown as follows:

Assume a factor translating quality into utility, *consumer sensitivity to quality* s_q :

$$u_q(q) = s_q \cdot q$$

Then the quadratic cost function to produce this level of consumer utility would be:

$$K_q(u) = \kappa_q \cdot u_q^2 = \kappa_q \cdot s_q^2 \cdot q^2$$

But as both κ_q and s_q^2 are *constant*, we can *aggregate* those two factors to a single factor, $k_q := \kappa_q \cdot s_q^2$, yielding:

$$K_q(q) = k_q \cdot q^2$$

Therefore, only *one* factor (k_q) is needed to relate produced consumer utility to cost.

[12] z_i^* therefore is the outcome of consumer *i*'s optimization (maximization of utility) with regard to one particular producer's offered locations.

Maximizing individual utility u_i , **one unit** will be purchased from the one producer j whose terms (λ_j, p_j, q_j) allow for the **maximum positive net surplus** u_{ij} . This can be at the nearest location, just as well as at a location further away, if quality and price over-compensate for distance (transportation costs).
As there is no outside good[13], a consumer will *not* purchase if his net surplus is negative with *every* producer's offer.

2.4.6 Range of attraction r_j

Consequently, and following from maximum consumer surplus (see the above sections) and transportation costs[14], every consumer i located within producer j's total **range of attraction** r_j

$$r_j(\lambda_j, p_j, q_j) = \lambda_j + 2 \cdot (q_j - p_j) / \tau \qquad [2.11]$$

will (be able and will actually) buy one unit, given that no competitor offers a higher net surplus.

[13] as in SALOP 1979

[14] see section 2.2.3 *Transportation cost factor* τ, as well as Table 2.1 *The profit function* $\pi(\lambda, p, q)$ *and its elements*

2.4.7 Total demand D_j

Boundaries and, again, competition aside, producer *j*'s **total demand D_j** is equal to (one unit times) consumer density *N/L*, times the purchasing characteristic space, consisting of horizontal expansion λ_j , plus those locations where consumers still buy because their utility exceeds price and transportation costs:[15]

$$D_j(\lambda_j, p_j, q_j) = (N/L) \cdot [\, \lambda_j + 2 \cdot (q_j - p_j) / \tau \,] \qquad [2.12]$$

[15] see also Table 2.1 *The profit function $\pi(\lambda, p, q)$ and its elements*

2.5 Logical model constraints

The following sections discuss those logical constraints that define the model's solution space defining the valid values for its strategic *variables*, i.e. horizontal expansion λ, quality q, and price p (see chapter 2.1 *Strategic variables*).
Analytical border and corner solutions (marginal solutions) will unfold along those same constraints.

For *parameter* constraints, i.e. non-negativity of parameters, see chapter 2.2.

2.5.1 Constraint 1: Non-negative maximum consumer surplus

In consequence of terms [2.7], [2.8], and [2.10] in sections 2.4.2 through 2.4.4, respectively, price p may never exceed quality q, in order to not let consumer surplus (within the boundaries of horizontal expansion λ) fall below zero, which would result in *no* consumers purchasing at all, be they in- or outside λ.

$$\text{C1:} \quad q - p \geq 0 \quad \Leftrightarrow \quad p \leq q \qquad [2.13]$$

Converging towards C1 implies a **maximum pricing strategy**, leaving no consumer surplus at all. Therefore, no consumer located outside the monopolist's horizontal expansion λ has an incentive to purchase, as transportation costs causes their surplus to drop below zero.

2.5.2 Constraint 2: Non-negative horizontal expansion

As all strategic variables, horizontal expansion λ (see section 2.1.2) cannot be negative:

$$\text{C2:} \quad \lambda \geq 0 \qquad [2.14]$$

2.5.3 Constraint 3: No excess horizontal expansion / excess range of attraction

Maximum horizontal expansion can be argued in terms of

$$\text{C3a:} \quad \lambda \leq L \qquad [2.15]$$

in order for *horizontal expansion* λ (see section 2.1.2) to not exceed characteristic space length L (see section 2.2.2), or as

$$\text{C3b:} \quad \lambda \leq L - 2/\tau \cdot (q - p) \qquad [2.16]$$

in order for *range of attraction r* (see section 2.4.6, i.e. term [2.11]) to not exceed L.

Clearly, C3b includes C3a, which is why they are subsumed in the section at hand. The solution space resulting from C3a (and C1 and C2 and C4) will include the one resulting from C3b, being identical only at C1 where q equals p and the difference term $-2/\tau \cdot (q - p)$ therefore equals 0.

Constraints are not necessary *logically*, as any y^* optimized in terms of profit *will* not exceed L, but, *analytically*, they will still prove useful (and, at times, pivotal).

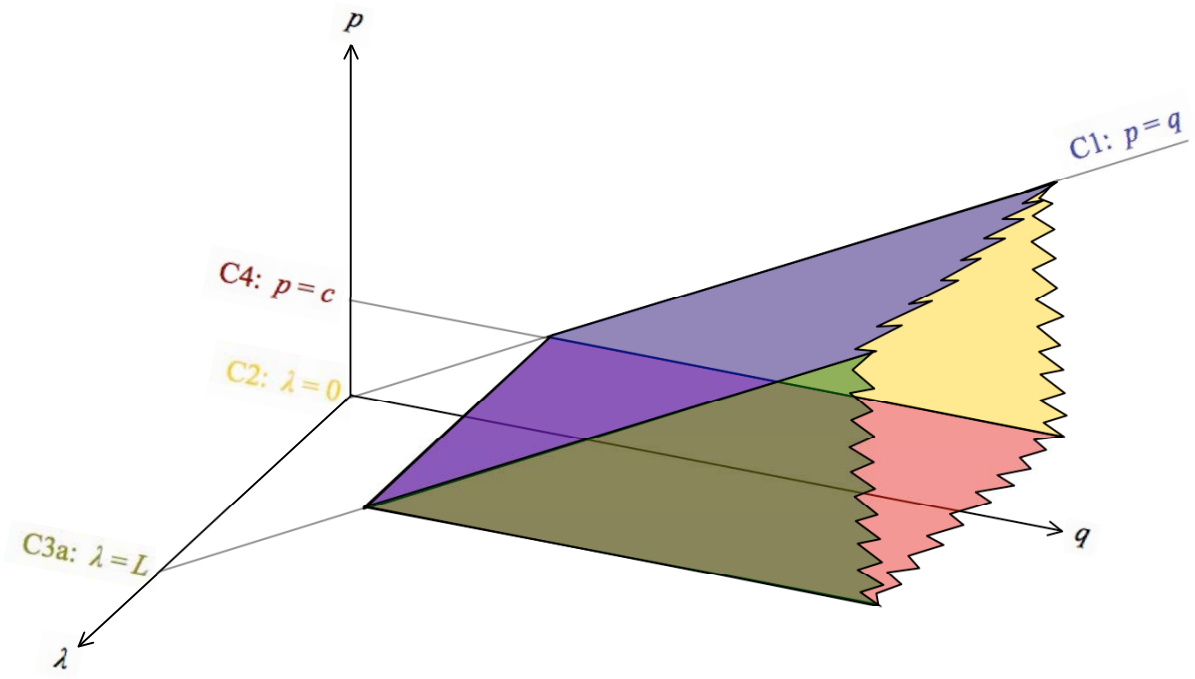
2.5.4 Constraint 4: Non-negative price margin

Price may not fall short of marginal cost, lest the producer incur a loss per unit sold.

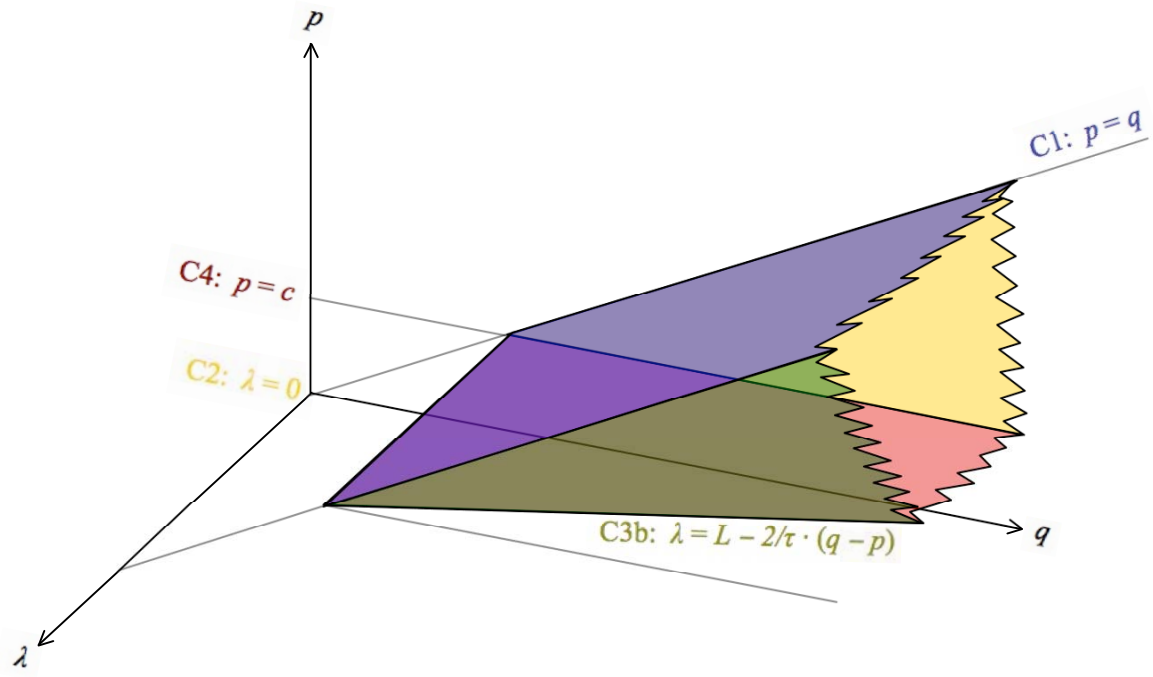
$$\text{C4:} \quad p \geq c \qquad [2.17]$$

2.5.5 Resulting solution space

All together, constraints C1 to C4 constitute solution spaces as outlined in the following Graphics 2.II and 2.III, respectively:



Graphics 2.II: Solution space constituted by logical constraints C1, C2, C3a, and C4



Graphics 2.III: Solution space constituted by logical constraints C1, C2, C3b, and C4

Again, it becomes clear that the solution space defined by constraint C3b is contained in the one defined by C3a. Thus, fulfilment of the stronger constraint C3b implies fulfilment of C3a.

3. PURE MONOPOLY

Chapter 3 discusses the case where the monopolist agent is *not* threatened by entry of one or more potential entrants.

3.1 Profit maximization

The monopolist will attempt to maximize profit. The corresponding optimization is shown in [3.1]:

$$\pi(\lambda, p, q) = (N/L) \cdot [\lambda + 2 \cdot (q - p)/\tau\ ] \cdot (p - c) - k_\lambda \cdot \lambda^2 - k_q \cdot q^2 \rightarrow \max! \quad [3.1]$$

The resulting solutions will be discussed in the following sections.

The **solution** that will turn out to be **optimal** is presented in section 3.1.5 *Marginal solution 3b: Maximum range of attraction*, but the other discussions may be of some interest, too, as they represent different **extreme strategies**:

- Marginal solution 1: Maximum pricing
- Marginal solution 2: Minimum horizontal expansion
- Marginal solution 3a: Maximum horizontal expansion
- Marginal solution 3b: Maximum range of attraction

- Marginal solution 4: Minimum pricing

But before we look at those extreme (marginal) strategies, the first-hand inner equilibrium (non-marginal) solution must be examined:

3.1.1 The non-marginal solution

It can be shown[16] that the analytical equilibrium (non-marginal) solution of problem [3.1] *can* be a maximum BUT it then necessarily violates the constraints of non-negativity of strategic variables. Therefore, it does **not** constitute a **valid solution**.

This leaves us with the marginal solutions (MS1 to MS4) unfolding along the logical model constraints C1 to C4 (discussed in sections 2.5.1 to 2.5.4 and outlined in Graphics 2.II and 2.III, respectively — see section 2.5.5 *Resulting solution space*), which will be discussed in the following sections 3.1.2 through 3.1.5.

3.1.2 Marginal Solution 1: Maximum price

Maximum price or Quality pricing (border condition [3.2]) is the marginal case of the **non-negative consumer surplus** constraint ($p \leq q$, cf. section 2.5.1):

$$\text{MS1:} \quad p = p_{max} = q \qquad [3.2]$$

The **maximum possible price** is charged, and **no consumer surplus** left. Therefore, no consumer located outside the monopolist's horizontal expansion λ, where he would have to incur actual transportation costs, has an incentive to buy, and will consequently not purchase.

The resulting (modified) optimization equals

[16] see Appendix A.1.3, p.59

$$\pi_{MS1}(\lambda, q = p) \;=\; (N/L)\;\lambda\;(q - c)\; - k_\lambda \cdot \lambda^2\; - k_q \cdot q^2 \;\rightarrow\; \max! \qquad [3.3]$$

It can be shown[17] that, depending on the parameter set, the critical point can indeed be a maximum, but it then necessarily violates the non-negativity constraints of strategic variables q^* and $p^* = q^*$. Therefore, **MS1** alone will **not** produce a **valid solution**.

Corner solutions at C2, C3a/C3b, and C4 may still be optimal and will be discussed in sections 3.1.3, 3.1.4, and 3.1.5, respectively.

3.1.3 Marginal Solution 2: Minimum horizontal expansion

Minimum or no horizontal expansion (border condition [3.4]) is the marginal case of **non-negative horizontal expansion** $\lambda \geq 0$ (cf. section 2.5.2):

$$\text{MS2:}\quad \lambda = \lambda_{min} = 0 \qquad [3.4]$$

The resulting optimization equals

$$\pi_{MS2}(p, q) = (N/L)\;\; 2/\tau \cdot (q - p)\,(p - c)\; - k_q\, q^2 \;\rightarrow\; \max! \qquad [3.5]$$

It can be shown[18] that a maximum can exist but results in a negative value for quality, and in a price below marginal cost c, thus violating model constraints. If the critical point is *not* a maximum, the optimum may be found among the corner solutions, but again it can be shown that **no** corner solution produces a **valid** (non-zero) **solution**.

[17] see Appendix A.1.4

[18] see Appendix A.1.5

3.1.4 Marginal Solution 3a: Maximum horizontal expansion

Maximum horizontal expansion (border condition [3.6]) is the marginal case of the **no excess horizontal expansion** condition (cf. conditions [2.15] in section 2.5.3):

$$\text{MS3a:} \quad \lambda = \lambda_{max} = L \qquad [3.6]$$

The modified optimization resulting from [3.6] equals

$$\pi_{MS3a}(p, q) = N \cdot (p - c) - k_q q^2 - k_\lambda L^2 \rightarrow \max! \qquad [3.7]$$

It becomes immediately clear that, for a profit-maximizing monopolist, it makes no sense to offer a quality exceeding price, as there are no consumers left outside our horizontal expansion λ_{max} to attract by offering a positive consumer surplus ($q - p$). As a consequence, MS3a **converges towards Constraint 1** ($p \leq q$) until $\boldsymbol{p^* = q^*}$, further reducing the problem to

$$\pi_{MS3a}(p) = N (p - c) - k_q p^2 - k_\lambda L^2 \rightarrow \max! \qquad [3.8]$$

the critical point of which necessarily is a profit maximum located at

$$p^* = q^* = N / 2k_q \qquad [3.9]$$

But as a comparison with **Marginal solution 3b** (outlined in the following section 3.1.5) will show that MS3b **is**, in general, **superior** to MS3a, we will move on to it.

The *other* corner solution at Constraint 4 ($p \leq c$) or, consequently, ($p = c$), can by contrast *not* be optimal, as, with no margin ($p - c$), it is naturally impossible to generate positive profit.

3.1.5 Marginal Solution 3b: Maximum range of attraction

Where in 3.1.4, horizontal expansion goes to the very limits of the consumer spectrum L regardless of marginal profit (which actually becomes negative once the *range of attraction r* (see section 2.4.6) has reached the limits of L), a marginal solution along the lines of constraint C3b (**no excess range of attraction**, see [2.16] in section 2.5.3) might prove more useful for optimizing profit:

$$\text{MS3b:} \quad r = \lambda + 2/\tau \cdot (q - p) =! \; r_{max} = L \quad \rightarrow$$
$$\lambda^*_{max} = L - 2/\tau \cdot (q - p) \qquad [3.10]$$

Thus, λ^*_{max} may be seen as the maximum *rational* horizontal expansion, as further expansion will neither increase the numbers of consumers nor profit, but only cause added expansion costs.

$$\pi_{MS3b}(p, q) = N \cdot (p - c) - k_q \, q^2 - k_\lambda \cdot [L - 2/\tau \cdot (q - p)]^2 \rightarrow \text{max!} \qquad [3.11]$$

The superiority of [3.11] over [3.7] becomes clear by comparing the expansion cost term, whose value is systematically lower by $2/\tau \cdot (q - p)$ than the corresponding cost term in [3.7]. Therefore, **MS3b is**, in general, **superior to MS3a**.

It can be shown[19] that the resulting optimum is indeed always an inner equilibrium **profit maximum** at

[19] as can be seen in Appendix A.1.7, with *any* strictly positive parameter configuration

$$q^* = N / 2k_q \quad [3.12]$$

$$p^* = (N / 2k_q + N\tau^2 / 8k_\lambda - L\tau/2) \quad [3.13]$$

$$\lambda^*_{max} = N\tau / 4k_\lambda \quad [3.14]$$

It is still possible for p^* and q^* to violate Constraint 1 ($p \leq q$, non-negative consumer surplus, see section 2.5.1).[20] In that *special* case, the constraint simply comes into effect, and the MS3b solution *converges* to the corner solution (MS1+3) of

$$p^* = q^* = N / 2k_q \quad [3.15]$$

With no consumer surplus ($q^* - p^*$), maximum attraction range (MS3b) *equals* maximum horizontal expansion (MS3a):

$$\lambda^* = \lambda_{max} = L \quad [3.16]$$

In view of *progressive* costs to cover increasing market shares (*quadratic* cost terms for both quality and expansion), the optimality of ***full* market coverage** may come as a surprise. It can be shown though, that even *lacking* the advantageous possibility to attract via expansion (e.g. due to infinitely high expansion costs: $k_\lambda = \infty$), full market coverage is *still* optimal!

For a detailed deduction, see Excursus: *Optimality of full market coverage without the possibility to expand* in Appendix A.1.7 on p. 75.

[20] this is true if $N/L > 4k_\lambda/\tau$

3.1.6 Marginal Solution 4: Minimum price

Minimum or Marginal cost pricing (border condition [3.17]) is the marginal case of the **non-negative price margin** condition ($p \geq c$, cf. condition [2.17] in section 2.5.4):

$$\text{MS4:} \quad p = p_{\min} = c \qquad [3.17]$$

With no margin and hence no income, the profit function collapses at MS4 to a mere costs function:

$$\pi_{MS4}(\lambda, q) = -k_\lambda \cdot \lambda^2 - k_q \cdot q^2 \qquad [3.18]$$

It is immediately clear that the only solution to [3.18] must be the **zero option** ($\lambda^* = 0$, $q^* = 0$). Therefore, all marginal solutions *combining* with MS4 to a corner solution are **non-valid**, too.

3.2 Conclusions

From ruling out the inner solution as well as the marginal solutions MS1, MS2, and MS4 in general, and by showing that MS3b is, in general, superior to MS3a,[21] we can conclude that Marginal Solution 3b, **maximizing range of attraction**, constitutes the **optimal strategy** for a monopolist not under threat of entry optimizing profit. [3.19] summarizes **optimal** non-zero **strategy**,

$$p^* = \min \left\{ \begin{matrix} q^* + N\tau^2/8k_\lambda - L\tau/2 \\ q^* \end{matrix} \right\}$$

$$q^* = N / 2k_q$$

$$\lambda^* = N\tau / 4k_\lambda \qquad [3.19]$$

[3.20] the **complete strategic set** (*complete* as it effectively covers *all* possible situations / parameter configurations, including the zero option if profit is negative):

$$q^* = \left\{ \begin{matrix} N/2k_q & \text{if } \pi^* \geq 0 \\ 0 & \text{if } \pi^* < 0 \end{matrix} \right\}$$

$$p^* = \left\{ \begin{matrix} \min \left\{ \begin{matrix} N/2k_q + N\tau^2/8k_\lambda - L\tau/2 \\ q^* \end{matrix} \right\} & \text{if } \pi^* \geq 0 \\ 0 & \text{if } \pi^* < 0 \end{matrix} \right\}$$

$$\lambda^* = \left\{ \begin{matrix} N\tau / 4k_\lambda & \text{if } \pi^* \geq 0 \\ 0 & \text{if } \pi^* < 0 \end{matrix} \right\} \qquad [3.20]$$

[21] unless MS3b *equals* MS3a, when p rises to the level of q (see section 2.5.1)

It is therefore *not* optimal for a monopolist to *restrain* activities (range of attraction) to some limited part of the market but to **spread over the entirety of the market**, in general not by horizontal expansion alone, but by employing a **mixed strategy** offering some horizontal expansion λ^* (dependent on expansion cost) as well as a positive maximum consumer surplus $q^* - p^*$ (dependent on quality production cost) attracting consumers (in fact, ***all* consumers**) located outside horizontal expansion.

In equilibrium, the degree of horizontal expansion λ^* depends only on expansion costs (k_λ) and, on the consumer side, transportation costs (τ) and the number of consumers (N).

Likewise, quality depends only on quality costs ($2k_q$) and consumers (N).

Price adjusts for the rest (i.e., characteristic space L).

As stated in 3.1.4, the optimality of ***full* market coverage** may come as a surprise in view of *progressive* costs to cover market shares (*quadratic* cost terms for both quality and expansion). It can be shown though, that even *lacking* the advantageous possibility to attract via expansion (e.g. due to infinitely high expansion costs: $k_\lambda = \infty$), full market coverage is *still* optimal!
For a detailed deduction, see *Excursus: Optimality of full market coverage without the possibility to expand* in Appendix A.1.7 on p. 75.

In terms of the exemplary interpretations discussed earlier, we could conclude that the mixed strategy (expansion plus quality) would include **product designs** that are *just* high-grade enough and *just* differentiated enough (e.g. in colour variants, or user interfaces suiting, up to a certain degree, both lovers of simplicity and high-end users who favour customizing every aspect) to be agreeable for *all* consumers, whatever their preferences may be. Real-world observations confirming this result can be found in abundance.

The following chapter 4 will examine *Entry Deterrence* strategies given *Threat of Entry*.

4. ENTRY DETERRENCE

The present chapter will discuss an incumbent's possible strategies of *entry deterrence* meant to repel a potential entrant. As *threat of entry* is given, the incumbent must anticipate entry in every conceivable way (*Central entry and competition, Central entry and peripheral competition,* and *Peripheral entry and competition*).

What will *not* be discussed in this context are *alternative strategies* on the part of the incumbent, like accommodating the entrant (e.g. by choosing a sideways location that would allow the entrant to profitably settle at some distance), which might well prove more profitable for the incumbent.[22]

Chapter 4.1 will give some general considerations in terms of entry deterrence, whereas chapter 4.2 will deal with the different strategies of entry on the part of the entrant and preemptive deterrence strategies in more detail.

4.1 Entry deterrence strategy in general

4.1.1 Central location

As the model assumes one single location per enterprise, and the potential entrant can be anticipated to move to the larger side of the market continuum (characteristic space L), it is rational for an incumbent wanting to deter the entrant to take a **central position at $L/2$**, thus

[22] see e.g. HOTELLING (1929)

equally covering both halves of L, and minimizing the larger "free" part of the market continuum.

Please note that this does, in general, *not* follow if the incumbent considers *accommodating* the entrant (which may well prove to be more profitable and may therefore, in a *complete* strategic set, be the strategy of choice).

4.1.2 Choice of strategic variables

The incumbent (henceforth indexed 1) will choose his strategic variables of horizontal expansion (λ_1), quality (q_1), and price (p_1) in a way that will reduce any potential entrant's profits to zero, be he entering/competing in the centre of characteristic space or in its periphery.

Should it be possible to prevent **all** conceivable entry and competition strategies (central and peripheral, respectively) **simultaneously**, then **preemption (entry deterrence)** is possible.

4.2 Strategies of Entry versus Entry Deterrence

Generally, the entrant can be anticipated to employ (one of) three different strategies of entry. They will be discussed in the following sections 4.2.1 through 4.2.3.

4.2.1 *Central entry and competition*

The first strategy of entry the incumbent (indexed 1) must anticipate the entrant (henceforth indexed 2) to potentially employ, and therefore must prevent if he wants to deter him, is what will be called *Central entry and competition*, during the course of which the entrant would settle at the very **same location** in the middle characteristic space as the incumbent, in order to simultaneously compete on both sides of their shared central location. The entrant will try to offer a *higher surplus* to consumers in order to take them over, which is in turn anticipated by the incumbent.

Please note that, unlike in the classical HOTELLING model, the incumbent and/or entrant will attract consumers on *both* sides, as, with horizontal expansion and vertical quality integrated into the model, there is no point in ruling out identical location: expansion and quality are *meant* to potentially attract consumers from *beyond* the competitor's location.

Having taken the same central location, competition will either take place in the very centre of characteristic space *L*, or in the periphery; the latter case will be discussed in section 4.2.2 *Central entry and peripheral competition.*

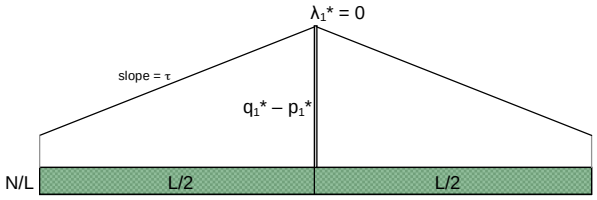
From the assumption of competition focussing on and the entrant locating himself in the very centre, and given that consumers purchase one unit maximum from the one producer that offers them the greater individual benefit, a situation *analogue to classical* ***BERTRAND competition***[23] results: One producer attracts *all* consumers, though not necessarily the one producer offering the *lowest price* as in the classical exogenous-quality model, but, as quality

[23] BERTRAND, Joseph Louis François (1883): "Théorie des Richesses: revue de *Théories mathématiques de la richesse sociale* par Léon WALRAS et *Recherches sur les principes mathématiques de la théorie des richesses* par Augustin COURNOT", *Journal des Savants* 67, pp. 499-508

has become variable and endogenous, the one offering the *highest level of maximum consumer surplus* ($q - p$) (see section 2.3.2), in this case: in the central location. Now as any potential entrant could emulate the incumbent's strategy except for offering an *incrementally higher level of maximum consumer surplus* ($q - p$), the incumbent is forced to ever increase his own offered level of consumer surplus ($q_1 - p_1$) for as long as his profits are non-negative, leaving him, in the end, with the **highest possible** level of **maximum consumer surplus** ($q_1^* - p_1^*$) that can be produced without incurring a loss (**no profit** condition [4.1]); only then (loss-free) **entry** is **foreclosed**.

$$\pi_1^* = 0 \qquad [4.1]$$

First of all, it becomes clear that if the highest possible level of **maximum consumer surplus** ($q_1^* - p_1^*$) alone is **sufficient**[24] **to cover the** entire **market** L, it is optimal to adopt a strategy of offering **zero horizontal expansion** (analogously to Marginal Solution 2 in section 3.1.3), as any strictly positive horizontal expansion $\lambda^* > 0$ would only cause costs, but would not lead to the attraction of more consumers — there simply aren't any who do not already purchase. Graphics 4.I illustrates the situation:



Graphics 4.I: Offered ($q_1^* - p_1^*$) sufficient to cover the entire market

[24] This is the case if $L \leq (N/2k_q - 2c)/\tau$ — that is, if consumer spectrum length L and/or transportation costs (factor τ) are sufficiently small. Note that, as a consequence, these two parameters are no longer factors in q^*, or p^*!

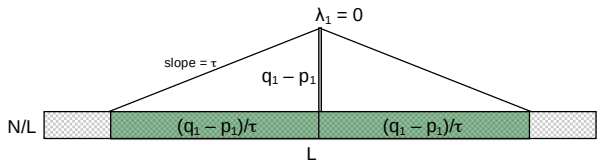
For this case, it can be shown that the optimal (non-zero)[25] strategy is:

$$\lambda_1^* = 0$$

$$p_1^* = c + N / 4k_q$$

$$q_1^* = N / 2k_q \qquad [4.2]$$

Here one might notice that the well-known limitation for p^* at q^* (non-negative maximum consumer surplus constraint, see section 2.5.1) do not show — it is, in this case, not necessary, as, if p^* grows near q^*,[26] consumer surplus will *no longer* be sufficient to cover the entire market, and the scenario discussed *next* becomes relevant:



Graphics 4.II: Offered ($q_1 - p_1$) *not* sufficient to cover the entire market

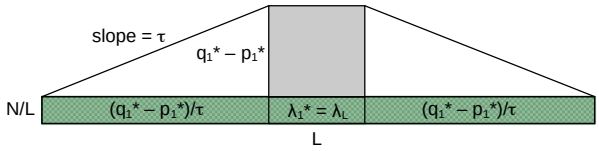
If ($q^* - p^*$) is ***not* sufficient**[27] **to cover the** entire **market** L (as illustrated in Graphics 4.II above), it can be shown that another strategy is optimal: In order to maximize ($q - p$), it can be shown[28] that it is optimal to offer a **positive horizontal expansion that exactly covers the entire market** ($\lambda_1^* = \lambda_L > 0$); thus, income from sales to *all* consumers can be invested into production of higher quality q_1 ; this is analogue to Marginal Solution 3b in section 3.1.5. The situation is illustrated in Graphics 4.III:

[25] given that the parameter configuration allows for non-negative profit — in this case of quasi BERTRAND competition, zero profit. If parameter configuration does result in strictly *negative* profit, well, then the zero option is of course preferable.

[26] This happens when $4c\ k_q$ grows close to N.

[27] This is the case if $L \leq (N/2k_q - 2c)/\tau$ — that is, if consumer spectrum length L and/or transportation costs (factor τ) are sufficiently small. Note that, as a consequence, these two parameters are no longer factors in q^*, or p^*!

[28] see Appendix A.2.1, p.83



Graphics 4.III: Positive horizontal expansion to cover the market ($\lambda_1^* = \lambda_L > 0$)

The (non-zero) strategic set is stated in [4.3]:

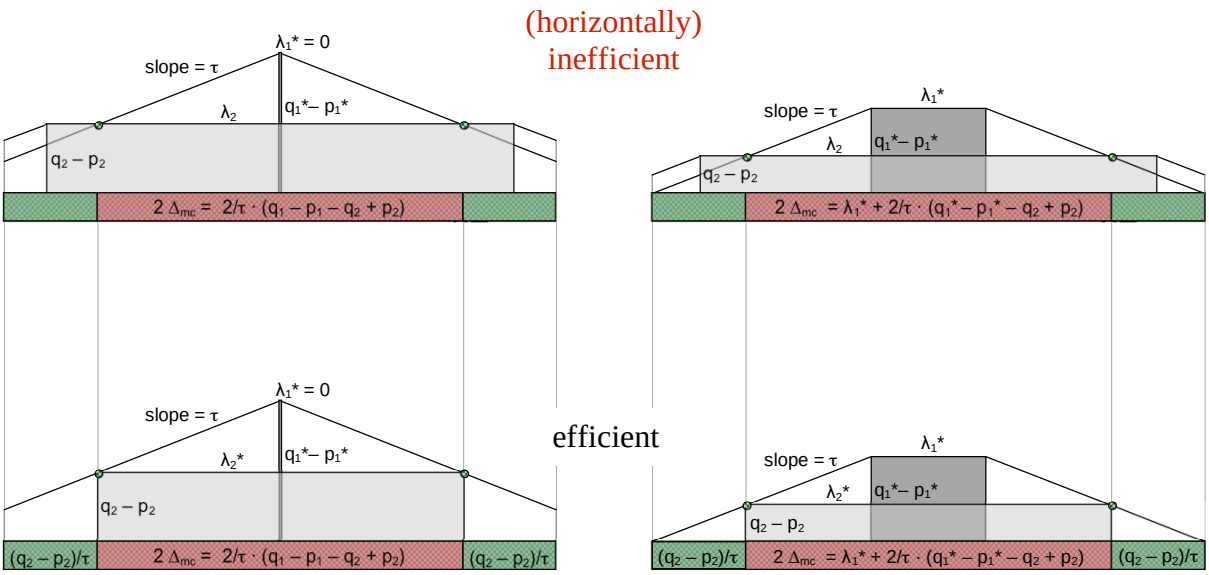
$$
\begin{aligned}
q_1^* &= N / 2k_q \\
p_1^* &= p_1^*(q_1^*) \\
&= {}^1/_8\, k_\lambda k_q \{N\tau^2 + 4\, k_\lambda N - 4\, k_\lambda k_q L\tau - \sqrt{[}\, k_q N\tau^2\, (k_q N\tau^2 + 4k_\lambda\, (N - 2k_q\, (2c + L\tau)))]\} \\
\lambda_1^* &= \lambda_1^*(p_1^*, q_1^*) \\
&= L - 2/\tau\, (q_1^* - p_1^*) \qquad [4.3]
\end{aligned}
$$

It follows from the very approach that in *both* cases ($(q - p)$ *sufficient* to cover the market; horizontal expansion optimized in order to *just* cover the market), **it is *impossible* for any potential entrant to offer a higher ($q - p$) at the central location** than an incumbent having renounced to the totality of his profits in order to maximize central consumer surplus himself.

Nota: (Total) pre-emption is only possible if the following alternative strategies of entry can be averted *simultaneously* (that is, *in addition* to preventing *Central entry and competition*). If a strategy preventing *all* approaches simultaneously does *not* exist, then neither does a NASH equilibrium, and systematic pre-emption is impossible.

4.2.2 *Central entry and peripheral competition*

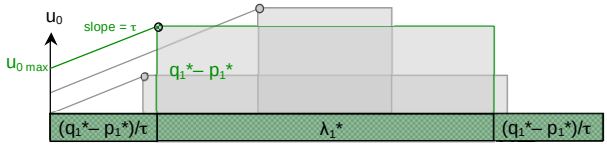
The second possibility for the entrant to enter the market is to take the very same central location as the incumbent (in order to equally reach customers located both on the right and left halves of spectrum *L*), but, instead of trying to outcompete the incumbent by offering a higher consumer surplus at the centre (which is impossible without incurring a loss, as the incumbent already has *maximized central consumer surplus*, incurring total loss of his profits), the entrant circumvents the incumbent's rigorous offer by offering a significantly lower ($q - p$), but by carrying this consumer surplus sideways into the periphery by expanding horizontally, thus "slipping under" the incumbent's offer (see Graphics 4.IV).



Graphics 4.IV: Efficiently reaching the marginal customer in terms of horizontal expansion λ_2

It is plausible and true that certain parameter configurations actually *do* allow for profitable *Central entry and peripheral competition.* Under those circumstances, a whole new competitive situation arises as the incumbent must anticipate profitable entry, and adapt his preemptive strategy accordingly:

The incumbent must, again, optimize his offer until profit is reduced to zero, as he must rule out strategic emulation (infinitesimal undercutting) by the entrant (BERTRAND competition). But now optimization no longer aims at *central* consumer surplus but at *peripheral* consumer surplus instead, as illustrated in Graphics 4.V:



Graphics 4.V: Maximizing peripheral consumer surplus u_0 in case of full market coverage (FMC)

The stable strategic set maximizing u_0 in case of full market coverage (FMC) is derived in Appendix A.2.2[29] and stated in [4.4]:

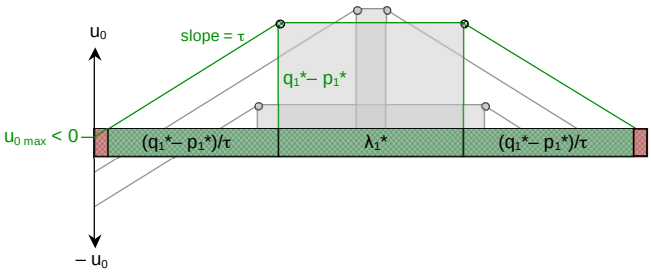
$$q_1^* = N / 2k_q$$

$$\lambda_1^* = N\tau / 4k_\lambda$$

$$p_1^* = c + N / 4k_q + N\tau^2 / 16k_\lambda \qquad [4.4]$$

In case of *incomplete* market coverage (as illustrated in Graphics 4.VI),

[29] see Appendix A.2.2 p.85



Graphics 4.VI: Maximizing peripheral consumer surplus u_0 in case of incomplete market coverage (IMC)

it can be shown[30] that an analytical solution for IMC exists, too, but that it

- *overreacts* to incomplete market coverage (IMC) conditions *by increasing maximum consumer surplus* $(q - p)$ *even further* (instead of reducing it, according to the lower income generated when not covering the whole market), thereby
- *re-establishing full market coverage* (thereby violating its own analytical assumptions), at the cost of
- *generating* increasingly huge *losses*, and
- taking place within the very *same parameter space*[31] *as the FMC solution* where it is
- *systematically dominated by the FMC solution* as the latter optimizes according to the correct (FMC) assumptions.

Therefore (as the IMC solution systematically creates FMC conditions, thereby violating its own assumptions), the **FMC solution** (optimizing on the correct assumption to begin with) **is dominant**.

[30] see Appendix A.2.2, pp. 91-93

[31] the same *valid* parameter space, that is: It can be shown that there actually *is* a parameter space where the IMC solution manifests itself in actual IMC space (outside of FMC space) – but there it produces (invalid) *negative values* for λ^*, p^*, and q^*.

It can furthermore be shown that an analytical **best response** to the strategic FMC set [4.4] exists, [32] but that **no** parameter configuration exists that would allow for non-negative profits; therefore, we conclude that **FMC strategy** [4.4] **is indeed preemptive**.

4.2.3 *Peripheral entry and competition*

Peripheral entry and competition will be discussed as a counter-strategy to both *Central entry and competition*, and *Central entry and peripheral competition.*

Either way, the potential entrant settles left or, equivalently, right of the incumbent's central location in order to avoid the pre-emptively high level of offered consumer surplus in the centre (see the above section 4.2.1 *Central entry and competition*). Transport costs arising between the chosen peripheral and the incumbent's central location *shield* the entrant's offer from the fierce competition formed by the incumbent's central offer, as transport costs systematically discount this maximized level of $(q - p)$ to a significantly *lower* level of *net* consumer surplus in peripheral locations that might still allow for entry and competition.

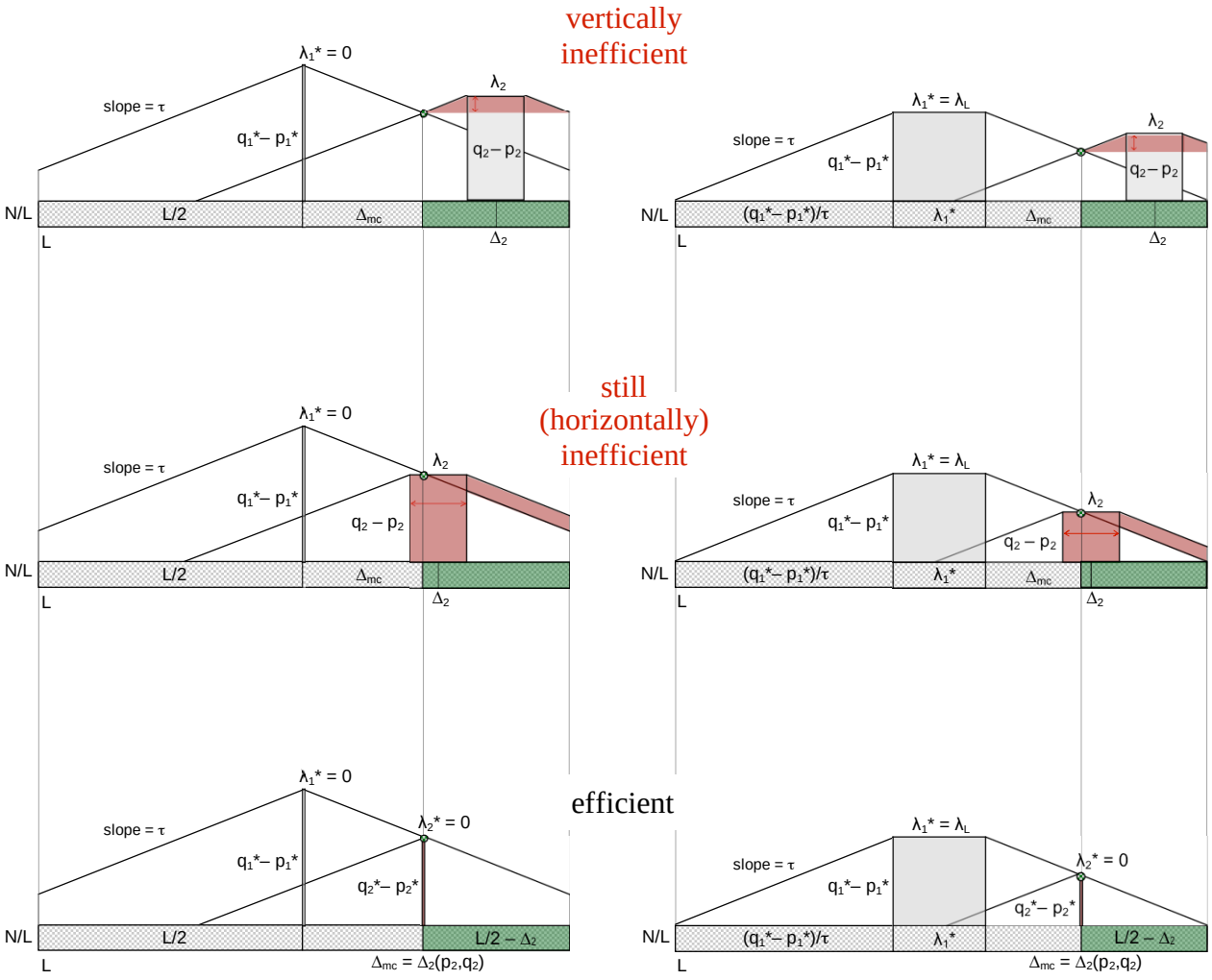
As the entrant's problem boils down to *reaching the marginal customer* (located at Δ_{mc}) by offering an incrementally higher net surplus (these points of *indifference* are indicated as green dots in the following Graphics 4.VII) at his location, and, as a consequence, serving *all* $(N \cdot (L/2 - \Delta_{mc}))$ customers located *beyond* the marginal customer's location, it becomes clear that it is *always*

- *inefficient* to offer a higher maximum consumer surplus than needed to target the marginal consumer ("vertically inefficient"), and, consequently, *inefficient* to locate so far out that the targeted marginal consumer would no longer be located *within* the entrant's horizontal expansion λ_2 but *between* the incumbent's and the entrant's expansion.

[32] see Appendix A.2.2

- *inefficient* to actually produce a positive horizontal expansion ("horizontally inefficient"), or, conversely, *efficient* to opt for no horizontal expansion at all ($\lambda_2^* = 0$);

these points are rational regardless of the incumbent offering horizontal expansion to cover the market ($\lambda_1^* = \lambda_L > 0$) or not ($\lambda_1^* = 0$), and regardless of where the entrant's actual (or, optimized) location (without loss of generality, at Δ_2 from the centre) might be:



Graphics 4.VII: Efficiently reaching the marginal customer: no horizontal expansion

It can be shown[33] that because of the fact that the entrant would need to provide for a competitive quality q_2 (or, more precisely, a competitive net consumer surplus $(q_2 - p_2 - \tau \cdot |z_i - z_i^*|)$[34] *without* being able to spread his *quality production costs* $(k_q \cdot q_2^2)$ over *both* halves of the consumer spectrum (generating income on both halves of L), it is **impossible for the entrant to reach the marginal customer** (via $(q_2^* - p_2^*)$) **without incurring a loss**, even if he tries to do so efficiently ($\lambda_2^* = 0$), and, naturally, whilst maximizing profits.

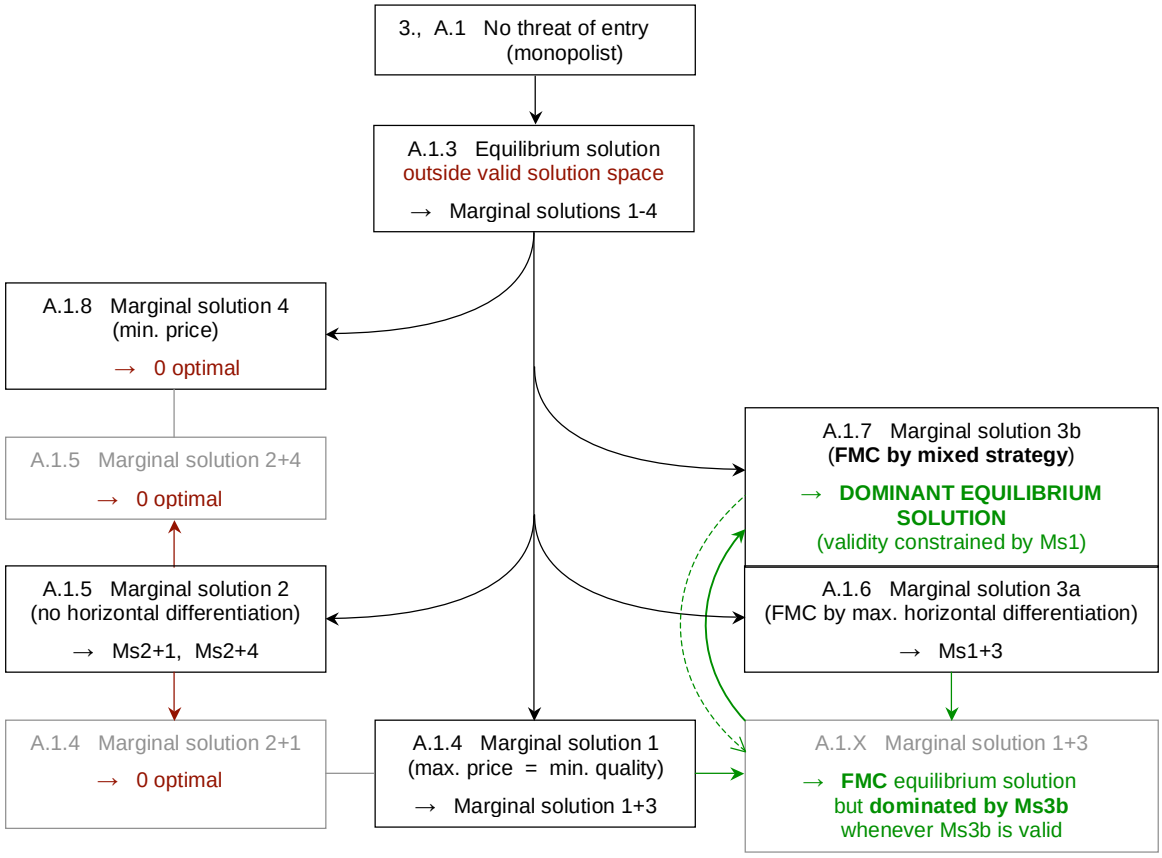
We can conclude that with global quality costs, it is vital to serve customers on *both* sides, as could be seen in 4.2.2 *Central entry and peripheral competition.*

[33] see Appendix A.2.3

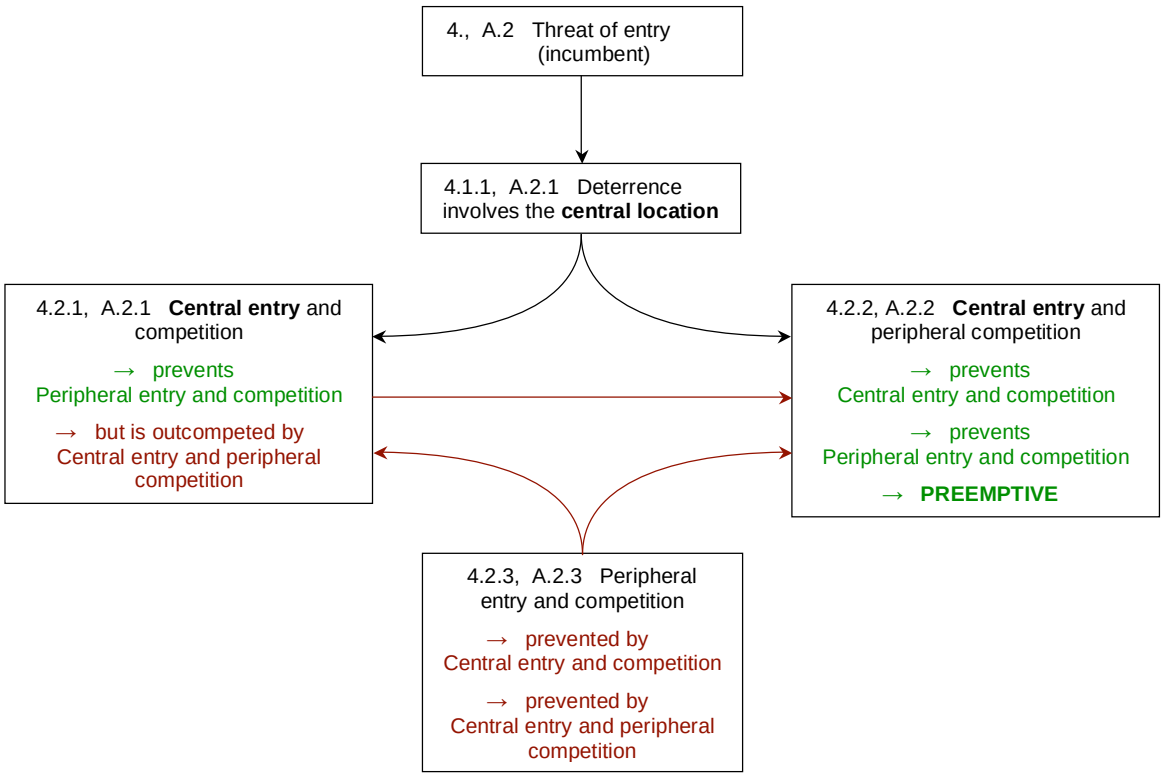
[34] see section 2.4.4 *Net consumer surplus*

5. CONCLUSIONS

Graphics 5.I and 5.II illustrate the analytical logic leading to the main conclusions from chapters 3 and 4, respectively, which are summarized in the following sections 5.1 through 5.5 :



Graphics 5.I: No threat of entry model solution scheme



Graphics 5.II: Threat of entry model solution scheme

5.1 Full market coverage (FMC) as a result from global quality costs

With global quality costs (e.g., highly variable research & development costs dominating quasi-invariant unit production costs), it is always optimal to cover the entire market — and never optimal to renounce to income in terms of purchases. Even the customers with the most exotic preferences (located at the extremes of the consumer spectrum) are served.

The operating-systems market may serve as an example: Global development costs are substantially higher than unit production costs; one product is (horizontally) differentiated to variants until they are suited for any application; (vertical) quality is maximized in order to prevent competition from establishing themselves at any location market.

5.2 Mixed strategy

Optimal strategies are always mixed strategies, be they conceived to maximize a non-threatened monopolist's profits, or to take preemptive effect as the optimal strategy of an incumbent under threat of entry: All optimal strategies employ both **horizontal differentiation**/variation of the basic product, and a surplus **vertical quality** (a positive maximum surplus / non-negative net surplus for *all* consumers, even) in order to attract the entire consumer spectrum.

As stated in 3.2, we could conclude that the optimality of above mixed strategy would favour **product designs** that are *just* high-grade enough and *just* differentiated enough (e.g. in colour variants, or user interfaces suiting, up to a certain degree, both lovers of simplicity and high-end users who favour customizing every aspect) to be agreeable for *all* consumers, whatever their preferences may be. Real-world observations confirming this result are abundant.

5.3 Central location

Optimal monopolist and entry-deterring strategies are **symmetrical**, employing a central location.

Please note that **asymmetrical** strategies can neither be ruled out for *accommodating* strategies, nor (theoretically) for entrants (were it not for the preemptive effect of the incumbent's entry deterrence).

5.4 Monopolist behaviour

If threat of entry is *not* imminent, a monopolist will maximize his profits by *exactly* covering the entire market, starting from a central location, with a mixed strategy including both horizontal differentiation and vertical quality (a positive maximum consumer surplus, even).

The complete strategic set has been laid out in [3.20].

5.5 Entry deterrence under threat of entry

If a potential entrant exists, he *can* indeed be deterred by competition analogue to classical **BERTRAND competition**, which forces the incumbent to preemptively annihilate his profits, else the entrant emulate (or infinitesimally undercut) his offer and replace him. As in BERTRAND competition, undercutting continues (or is anticipated, rather) until an agent finally annihilates the entire profit.

Furthermore, it could be shown that the functioning **deterrence strategy** (maximizing peripheral consumer surplus up to the complete annihilation of profits) does not only prevent BERTRAND emulation, but also takes **preemptive effect** in view of attempts to enter the market employing an altogether *different* strategy, be it a symmetrical one deployed from the same central location (*Central entry and peripheral competition*), or an asymmetrical one where the entrant would take a peripheral location (*Peripheral entry and competition*).

5.6 Prospect

Future research could strive to develop a *complete* strategic set for the incumbent, including strategies other than entry deterrence, like accomodating the entrant, or tacit collusion. A comparison of all available strategies would entail the identification of a generally optimal strategy.

Analogue to SALOP (1979), the number of firms entering a market, given above strategic behaviour, might be determined.

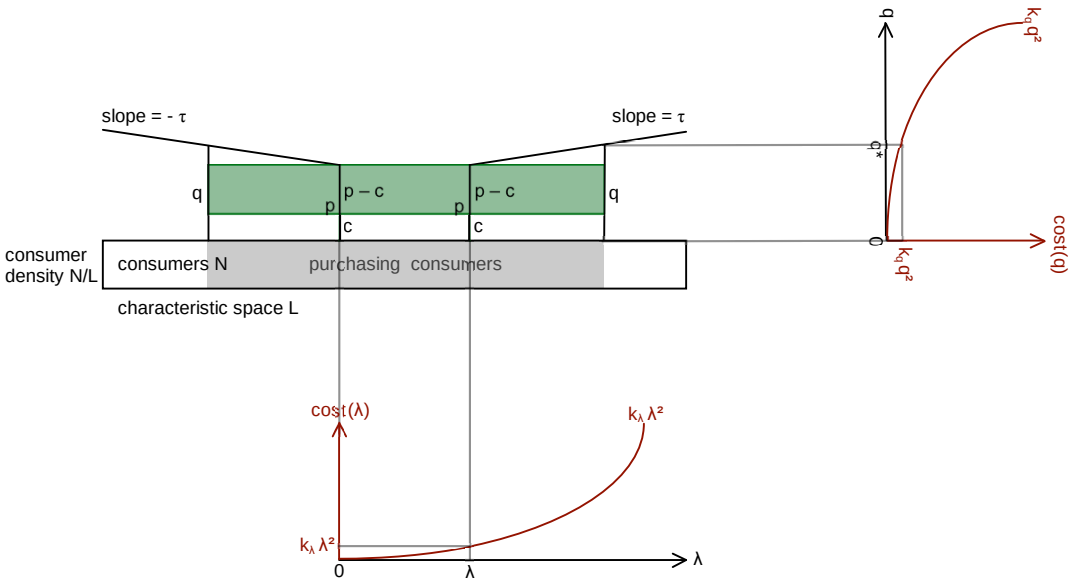
The different strategies, i.e. the one to be chosen in oligopolistic competition, may be compared with respect to welfare, including an analysis of the chosen vs. the socially optimal strategic variables (location, expansion, quality, and price) as well as the resulting number of firms.

A. APPENDIX

Appendices A.1 and A.2 will show the calculations for monopoly and entry deterrence, respecticely, on which statements in chapters 3. *Pure Monopoly* and 4. *Entry Deterrence* are based

A.1 Closed-form solution for the monopoly situation

A.1.1 The profit function



Graphics A.I: Interaction of the elements of the profit function $\pi(\lambda, p, q)$

The **profit function** is given by

$$\pi(\lambda, p, q) = N/L \ (\lambda + 2/\tau\,(q - p)) \cdot (p - c) - k_q \cdot q^2 - k_\lambda \cdot \lambda^2 \rightarrow \max !$$

subject to

Constraint C1: $p - q \leq 0$

Constraint C2: $-\lambda \leq 0$

Constraint C3: $\lambda + 2\,(q - p)/\tau \leq L$

Constraint C4: $-p + c \leq 0$

Constraint C5: $-q \leq 0$

A.1.2 The critical point

First, the unconstrained inner optimum is determined, in order to see if it is a maximum, and if it lies within the boundaries drawn by constraints C1-5.
It can be shown that an **inner maximum** with positive profit $\pi^*(p^*, q^*, \lambda^*)$ exists but *necessarily* **violates** the **non-negativity conditions** for λ^* and q^* (C2 and C5) as well as the marginal costs constraint C4 ($p^* \geq c$) :

First order conditions are:

$$\partial\pi/\partial p = N/L \ [\ \lambda + 2/\tau \ (q - 2\,p + c)\ ] = 0$$

$$\partial\pi/\partial q = -2\,k_q\,q + 2\,p\,N/L\tau - 2\,c\,N/L\tau = 0$$

$$\partial\pi/\partial\lambda = N/L\,(p - c) - 2\,k_\lambda\,\lambda = 0$$

Using Cramer's Rule

$$x_i = \det H_i / \det H \quad \text{for } \forall i$$

to solve the equivalent 3 x 3 linear equations system,

$$\begin{pmatrix} -4\,N/L\tau & 2\,N/L\tau & N/L \\ 2\,N/L\tau & -2\,k_q & 0 \\ +\,N/L & 0 & -2\,k_\lambda \end{pmatrix} \begin{pmatrix} p \\ q \\ \lambda \end{pmatrix} = \begin{pmatrix} -2\,cN/L\tau \\ 2\,cN/L\tau \\ cN/L \end{pmatrix}$$

$$\det \mathrm{H} = \begin{vmatrix} -4\,N/L\tau & 2\,N/L\tau & N/L \\ 2\,N/L\tau & -2\,k_q & 0 \\ N/L & 0 & -2\,k_\lambda \end{vmatrix} = -16\,k_q\,k_\lambda\,N/L\tau + 2\,k_q\,N^2/L^2 + 8\,k_\lambda\,N^2/L^2\tau^2$$

$$\det \mathrm{H_p} = \begin{vmatrix} -2\,cN/L\tau & 2\,N/L\tau & N/L \\ 2\,cN/L\tau & -2\,k_q & 0 \\ cN/L & 0 & -2\,k_\lambda \end{vmatrix} = -8\,c\,k_\lambda\,k_q\,N/L\tau + 2\,c\,k_q\,N^2/L^2 + 8\,c\,k_\lambda\,N^2/L^2\tau^2$$

$$\det \mathrm{H_q} = \begin{vmatrix} -4\,N/L\tau & -2\,cN/L\tau & N/L \\ 2\,N/L\tau & 2\,cN/L\tau & 0 \\ N/L & cN/L & -2\,k_\lambda \end{vmatrix} = 8\,c\,k_\lambda\,N^2/L^2\tau^2$$

$$\det \mathrm{H_\lambda} = \begin{vmatrix} -4\,N/L\tau & 2\,N/L\tau & -2\,cN/L\tau \\ 2\,N/L\tau & -2\,k_q & 2\,cN/L\tau \\ N/L & 0 & cN/L \end{vmatrix} = 4\,c\,k_q\,N^2/L^2\tau$$

we get an inner optimum at

$$p^* = \det \mathrm{H_p} / \det \mathrm{H} = c + 4\,c\,k_q\,k_\lambda\,L\,\tau\,/(k_q\,N\,\tau^2 + 4\,k_\lambda\,N - 8\,k_q\,k_\lambda\,L\,\tau)$$

$$q^* = \det \mathrm{H_q} / \det \mathrm{H} = +4\,c\,k_\lambda\,N\,/(k_q\,N\,\tau^2 + 4\,k_\lambda\,N - 8\,k_q\,k_\lambda\,L\,\tau)$$

$$\lambda^* = \det \mathrm{H_\lambda} / \det \mathrm{H} = +2\,c\,k_q\,N\,\tau\,/(k_q\,N\,\tau^2 + 4\,k_\lambda\,N - 8\,k_q\,k_\lambda\,L\,\tau)$$

A.1.3 Invalidity of the critical point

Seeing that the resulting profit equals,

$$\pi^*(p^*, q^*, \lambda^*) = -4\, c^2\, k_q\, k_\lambda\, N / (k_q\, N\, \tau^2 + 4\, k_\lambda\, N - 8\, k_q\, k_\lambda\, L\, \tau)$$

we can already conclude that it is impossible for π^* to be positive if constraints C2, C4, and C5 ($\lambda^* \geq 0$, $p^* \geq c$, and $q^* \geq 0$, respectively), as the same denominator $(k_q\, N\, \tau^2 + 4\, k_\lambda\, N - 8\, k_q\, k_\lambda\, L\, \tau)$ is contained in the terms for p^*, q^*, and λ^*, on the one hand, and π^* on the other hand, while the numerators' algebraic signs are obviously *not* the same.
Clearly, as inactivity would yield a better result ($\pi = 0$) than profit at the critical point ($\pi^* = 0$), the critical point cannot be a maximum. In order to determine it, we will have to systematically compare results at the constraints.

To determine if a maximum exists, we use *Sylvester's Criterion:*

If the Hessian's principal minors (the successive determinants $\det H_{i,i} < 0$) exhibit alternating algebraic signs (starting with a negative $\det H_{1,1} < 0$), then the function is negative definite, indeed producing a maximum:

$$H(\lambda, q, p) = \begin{pmatrix} -2k_\lambda & 0 & N/L \\ 0 & -2k_q & 2N/L\tau \\ N/L & 2N/L\tau & -4N/L\tau \end{pmatrix}$$

$$\det H_{1,1} = -2k_\lambda < 0$$

$$\det H_{2,2} = (-2k_\lambda)(-2k_q) = 4k_\lambda k_q > 0$$

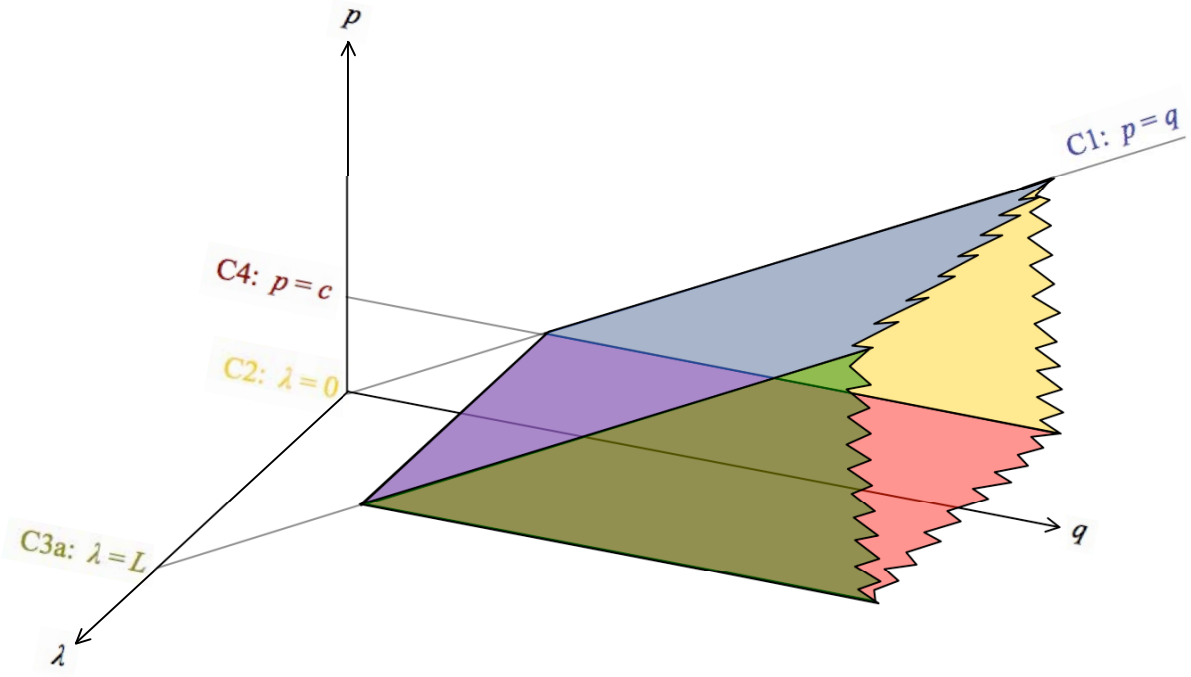
$$\det H_{3,3} = (2N/L^2\tau^2) \cdot (k_q N \tau^2 + 4k_\lambda N - 8k_q k_\lambda L \tau)$$

$$\det H_{3,3} < 0 \Leftrightarrow (k_q N \tau^2 + 4k_\lambda N - 8k_q k_\lambda L \tau) < 0 \Leftrightarrow \lambda^* < 0,\ p^* < c,\ q^* < 0,\ \pi^* > 0$$

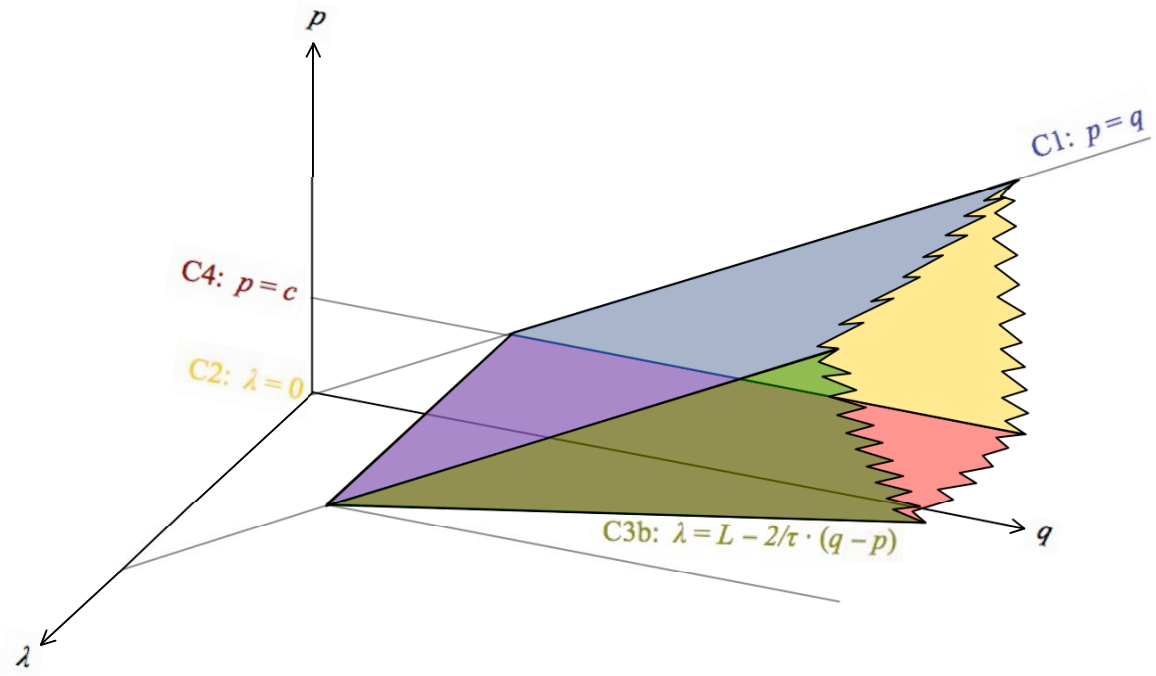
A maximum only exists outside the boundaries drawn by constraints C2, C4, and C5 for p^*, q^*, and λ^*; therefore, the solution to the problem (a valid maximum) must be found along one or more of the binding constraints C1-5.

Therefore, we can conclude that

- a **maximum** exists if $(k_q N \tau^2 + 4k_\lambda N - 8k_q k_\lambda L \tau)$ is negative,
- with **positive profit** equal to the constant term,
- but the corresponding critical point (the strategic set analytically maximizing profit) is then located **outside valid solution space** ($p^* < c$, $q^* < 0$, $\lambda^* < 0$).
- Having ruled out the inner solution, the **optimal solution must necessarily coincide with one of the marginal solutions** located at at least one of the variables' constraints (at the very boundaries of valid solution space):



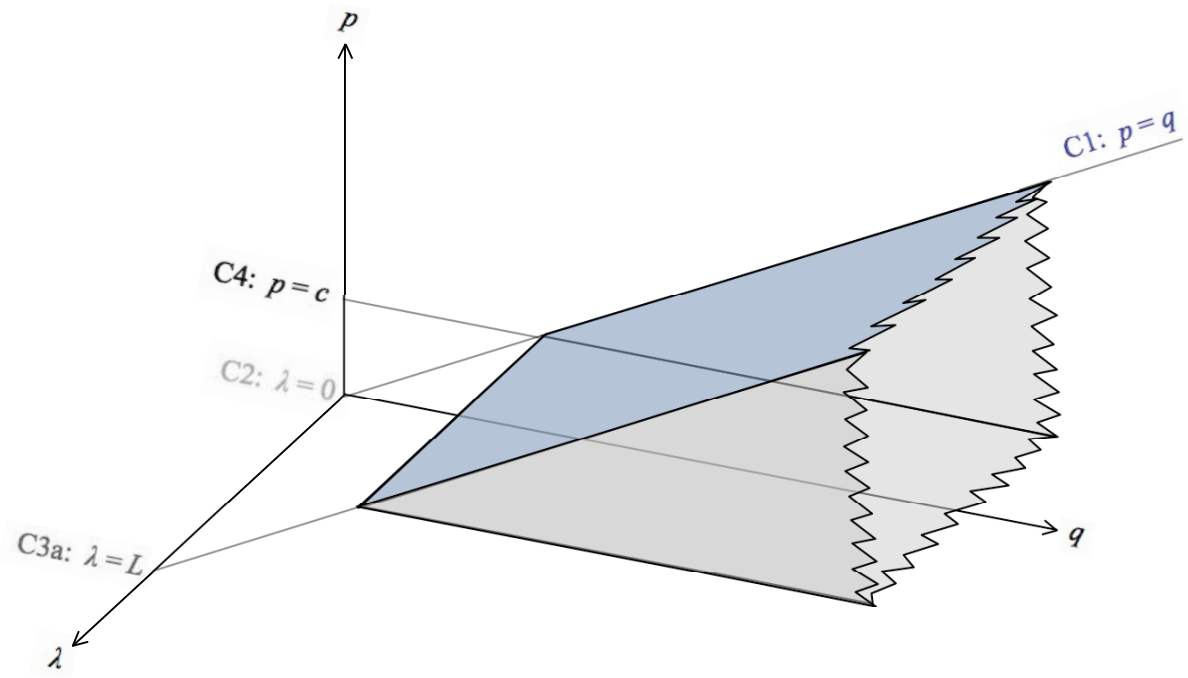
Graphics A.II: Solution space constituted by constraints C1, C2, C3a, and C4



Graphics A.III: Solution space constituted by constraints C1, C2, C3b, and C4

A.1.4 Marginal solution 1: Maximum price ($p = q$)

Constraint 1 states that price may not exceed quality, else consumer surplus be negative. The corresponding marginal solution is, therefore, a (maximum) price set equal to quality: $p = q$.



Graphics A.IV: Marginal solution 1: Maximum price ($p = q$)

The original profit function

$$\pi(\lambda, p, q) = N/L \ (\lambda + 2/\tau\,(q - p)) \cdot (p - c) - k_q \cdot q^2 - k_\lambda \cdot \lambda^2 \rightarrow \max !$$

is thereby reduced to

$$\pi(\lambda, q) = N/L\ (\lambda) \cdot (q - c) - k_q \cdot q^2 - k_\lambda \cdot \lambda^2 \rightarrow \max !$$

First order conditions are

$$\partial\pi/\partial\lambda = N/L\ (q - c) - 2\ k_\lambda\ \lambda = 0$$
$$\partial\pi/\partial q = \lambda\ N/L - 2\ k_q\ q = 0$$

Using Cramer's Rule to solve the 2 x 2 linear equations system,

$$\begin{bmatrix} -2\ k_\lambda & N/L \\ N/L & -2\ k_q \end{bmatrix} \begin{bmatrix} \lambda \\ q \end{bmatrix} = \begin{bmatrix} c\ N/L \\ 0 \end{bmatrix}$$

$$\det H = \begin{vmatrix} -2\ k_\lambda & N/L \\ N/L & -2\ k_q \end{vmatrix} = 4\ k_\lambda\ k_q - N^2/L^2$$

$$\det H_\lambda = \begin{vmatrix} c\ N/L & N/L \\ 0 & -2\ k_q \end{vmatrix} = -2\ c\ k_q\ N/L$$

$$\det H_q = \begin{vmatrix} -2\ k_\lambda & c\ N/L \\ N/L & 0 \end{vmatrix} = -2\ c\ N^2/L^2$$

we get an inner optimum at

$$\lambda^* = \det H_\lambda / \det H = -2\ c\ k_q\ L\ N\ /(4\ k_\lambda\ k_q\ L^2 - N^2)$$
$$q^* = p^* = \det H_q / \det H = -2\ c\ N^2\ /(4\ k_\lambda\ k_q\ L^2 - N^2)$$

with a critical profit of

$$\pi^*(p^* = q^*, \lambda^*) = + 4\, c^2\, k_q\, N^2 / (4\, k_q\, k_\lambda\, L^2 - N^2)$$

Again, we can already see that it is impossible for π^* and $p^* = q^*$, λ^* to be positive at the same time. If the Hessian's principal minors ($\det H_{1,1} = -2\,k_\lambda$, $\det H_{2,2} = 4\,k_q\,k_\lambda\,L^2 - N^2$) have alternating signs, then critical profit is positive and a maximum, but $p^* = q^*$ as well as λ^* are negative.

Therefore, we can conclude that

- a **maximum** exists if $(4\, k_q\, k_\lambda\, L^2 - N^2)$ is positive,
- with **positive profit**,
- but the corresponding **critical point** is located **outside valid solution space** ($p^* = q^* < 0$, $\lambda^* < 0$).
- Therefore, a valid solution at Constraint 1 must, if it exists, necessarily be a **corner solution** and coincide with more constraints than the one presently discussed.

The three marginal solutions limiting the solution space of Marginal solution 1 (see Graphics A.II) are

- **MS 1+2: Max. price ($p = q$) + min. horizontal expansion ($\lambda = 0$)**

  It is easy to see that with neither consumer surplus ($q - p = 0$) nor horizontal expansion ($\lambda = 0$), no consumers purchase, and the profit function collapses to a mere costs function

  $$\pi(q) = -k_q \cdot q^2 \rightarrow \max !$$

  whose optimal solution is, obviously, the zero option ($\boldsymbol{q^* = 0}$).

- **MS 1+4: Minimum = maximum price ($p = q = c$)**

  With no profit margin ($p - c$), the profit function again collapses to a mere costs function

  $$\pi(\lambda) = -k_q \cdot c^2 - k_\lambda \cdot \lambda^2 \rightarrow \max !$$

  and the zero option ($\boldsymbol{\lambda^* = 0}$) becomes optimal.

- **MS 1+3: Max. price ($p = q$) + max. horizontal expansion ($\lambda = L$)**

  Covering the whole market by one's offered horizontal expansion reduces the profit function to a convex quadratic function in q :

  $$\pi(q) = N \cdot (q - c) - k_q \cdot q^2 - k_\lambda \cdot L^2 \rightarrow \max !$$

  that allows for a simple F.O.C.

$\partial\pi(q)/\partial q \;=\; -\,2\,k_q\,q + N \;=\; 0$

resulting in the critical point of

$\boldsymbol{q^* \;=\; N\,/\,2k_q}$

which necessarily is a **maximum** as the S.O.C.

$\partial^2\pi(q)/\partial q^2 \;=\; -\,2\,k_q\,q \;<\; 0$

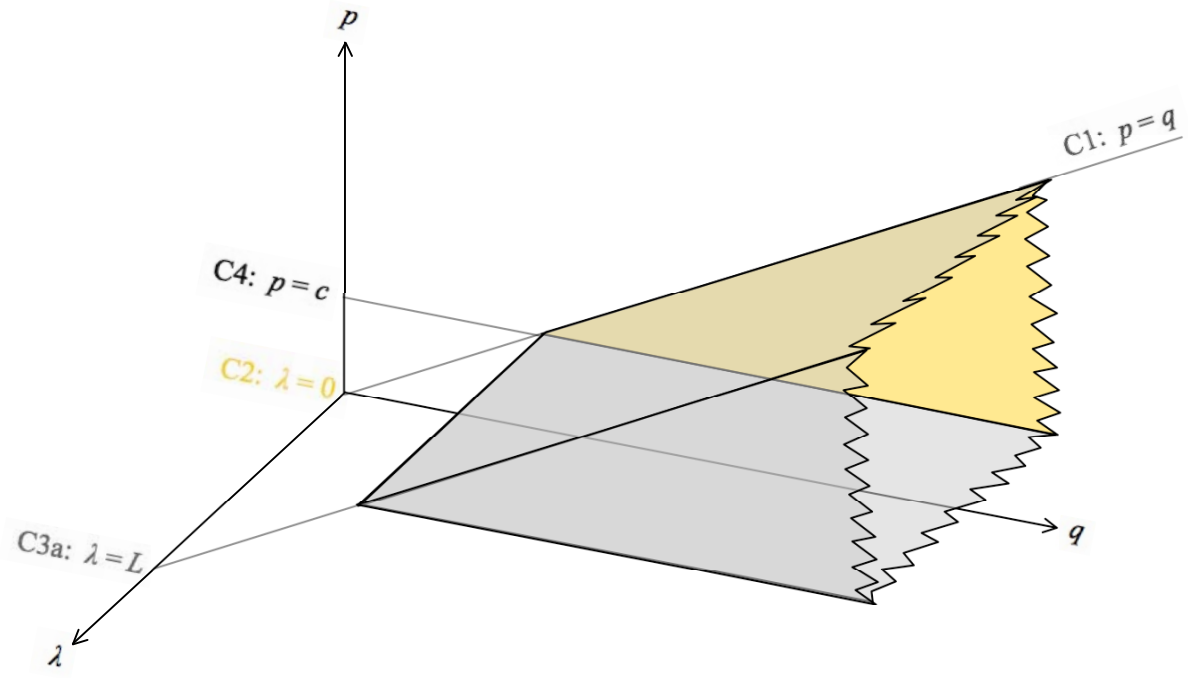
shows, where profits can be positive (naturally, that depends on the parameter set):

$\pi(q^*) \;=\; N^2/4k_q \;-c\,N\;-k_\lambda\,L^2$

Note: This valid solution (marginal solution at both Constraint 1 (*p* = *q*) and Constraint 3 (*λ* = *L*) needs to be compared to *all other possibly optimal solutions* before a statement on optimality can be made!

A.1.5 Marginal solution 2: Minimum horizontal expansion ($\lambda = 0$)

Constraint 2 states that horizontal expansion must be non-negative ($\lambda \geq 0$); the corresponding marginal solution hence involves zero horizontal expansion ($\lambda = 0$).



Graphics A.V: Marginal solution 2: Minimum horizontal expansion ($\lambda = 0$)

The original profit function

$$\pi(\lambda, p, q) = N/L\ [\lambda + 2/\tau\ (q - p)] \cdot (p - c)\ - k_q \cdot q^2\ - k_\lambda \cdot \lambda^2\ \rightarrow \max !$$

is, accordingly, reduced to

$$\pi(p, q) = N/L\ [2/\tau\ (q - p)] \cdot (p - c)\ - k_q \cdot q^2\ \rightarrow \max !$$

First order conditions are

$$\partial\pi/\partial p \;=\; N/L\;\;(2/\tau)\;\;(q - 2p + c) \;=\; 0 \quad\Leftrightarrow\quad -4\,N/L\tau\;\;p \;+\; 2\,N/L\tau\;\;q \;=\; -2\,cN/L\tau$$

$$\partial\pi/\partial q \;=\; N/L\;\;(2/\tau)\cdot(p - c) \;-2\,k_q\,q \;=\; 0 \quad\Leftrightarrow\quad 2\,N/L\tau\cdot p \;-2\,k_q\,q \;=\; 2\,cN/L\tau$$

Using Cramer's Rule to solve the 2x2 linear equations system,

$$\begin{bmatrix} -4\,N/L\tau & 2\,N/L\tau \\ 2\,N/L\tau & -2\,k_q \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} -2\,cN/L\tau \\ 2\,cN/L\tau \end{bmatrix}$$

$$\det H \;=\; \begin{vmatrix} -4\,N/L\tau & 2\,N/L\tau \\ 2\,N/L\tau & -2\,k_q \end{vmatrix} \;=\; 8\;k_qN/L\tau \;-\; 4\,N^2/L^2\tau^2$$

$$\det H_p = \begin{vmatrix} -2\,cN/L\tau & 2\,N/L\tau \\ 2\,cN/L\tau & -2\,k_q \end{vmatrix} \;=\; 4\,c\,k_q\,N/L\tau \;-\; 4\,cN^2/L^2\tau^2$$

$$\det H_q \;=\; \begin{vmatrix} -4\,N/L\tau & -2\,cN/L\tau \\ 2\,N/L\tau & 2\,cN/L\tau \end{vmatrix} \;=\; -8\,cN^2/L^2\tau^2 \;+\; 4\,cN^2/L^2\tau^2$$

we get an inner optimum at

$$p^* \;=\; \det H_p\,/\,\det H \;=\; c\,(N - k_q\,L\,\tau)/(N - 2\,k_q\,L\,\tau) \;=\; c \;+\; (c\,k_q\,L\,\tau)/(N - 2\,k_q\,L\,\tau)$$

$$q^* = p^* = \det H_q\,/\,\det H \;=\; c\;\;N \qquad /(N - 2\,k_q\,L\,\tau)$$

It can be seen that if $(N - 2\,k_q\,L\,\tau) > 0$, then $q^* > 0$, $p^* > 0$, $q^* > p^*$, and $p^* > c$.

Critical profit equals

$$\pi^*(p^*, q^*) = -c^2 k_q N / (N - 2 k_q L \tau)$$

$$\det H_{1,1} = -4N/L\tau < 0$$

$$\det H_{2,2} = 8 k_q N/L\tau - 4 N^2/L^2\tau^2 > 0 \quad \Leftrightarrow \quad (N - 2 k_q L \tau) < 0$$

Again, we see that it is impossible for π^* and p^* and q^* to be positive at the same time. If $(N - 2 k_q L \tau) < 0$, then the Hessian's principal minors ($\det H_{1,1}$ and $\det H_{2,2}$) have alternating signs, critical profit is positive and a maximum, but then q^* is negative and $p^* < c$.

Therefore, we conclude that

- a **maximum** exists if $(N - 2 k_q L \tau)$ is negative,
- with **positive profit** equal to the constant term,
- but the corresponding **critical point** (the strategic set analytically maximizing profit) is then located **outside valid solution space** ($q^* < 0$, $p^* < c$).
- Therefore, a **valid solution** at Constraint 2 must, if it exists, necessarily be a **corner solution** and coincide with more constraints than the one at hand.

The two marginal solutions limiting Marginal solution 2 (see Graphics A.II) are:

- **MS 2+1: Min. horizontal expansion ($\lambda = 0$) + max. price ($p = q$)**

  Discussed in above section A.1.4. No incentive to purchase, therefore no purchases, and no income. The optimal strategy is the **zero option** ($p^* = q^* = 0$).

- **MS 2+4: Min. horizontal expansion ($\lambda = 0$) + min. price ($p = c$)**

  Without a positive profit margin ($p - c$), the profit function collapses to a mere costs function

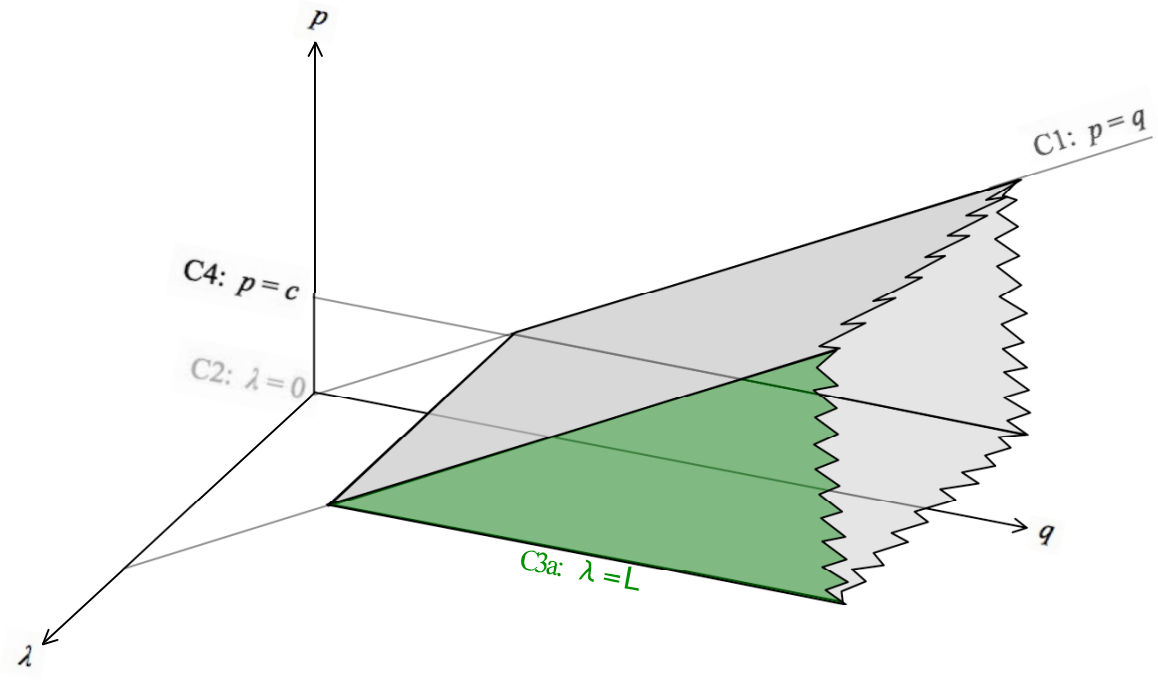
  $\pi(q) = -k_q \cdot q^2 \rightarrow \max !$

  and the **zero option** ($q^* = 0$) becomes optimal once again.

We can therefore conclusively rule out a valid solution at Constraint 2.

A.1.6 Marginal solution 3a: Maximum horizontal expansion ($\lambda = L$)

Constraint 3a states that it is not rational for horizontal expansion to exceed the consumer spectrum: $\lambda \leq L$. Please note that, other than Constraint 3b, the logic of this constraint remains unchanged even under threat of entry.



Graphics A.VI: Marginal solution 3a: Maximum horizontal expansion ($\lambda = L$)

With maximum horizontal expansion, the profit function is reduced to

$$\pi(p, q) = N \cdot (p - c) - k_q \cdot q^2 - k_\lambda \cdot L^2 \rightarrow \max !$$

Furthermore, as there is no market left to attract outside the boundaries of λ_{max}, it makes no sense to produce a higher quality q than necessary to generate price p. Thus, MS 3a necessarily converges towards MS 1 ($p \leq q$) until $p = q$. This case

$$\pi(p) = N \cdot (p - c) - k_q \cdot p^2 - k_\lambda \cdot L^2 \rightarrow \max !$$

has been discussed in chapter A.1.4, i.e. in section *MS 1+3* […],[35] leading us to a profit maximum at

$$p^* = q^* = N / 2k_q$$

Please note again: This solution needs to be **compared to all other valid solutions** (esp. Marginal solution 3b, see below) in order to be able to pronounce a generally valid statement on strategic optimality.

A.1.7 Marginal solution 3b: Full market coverage

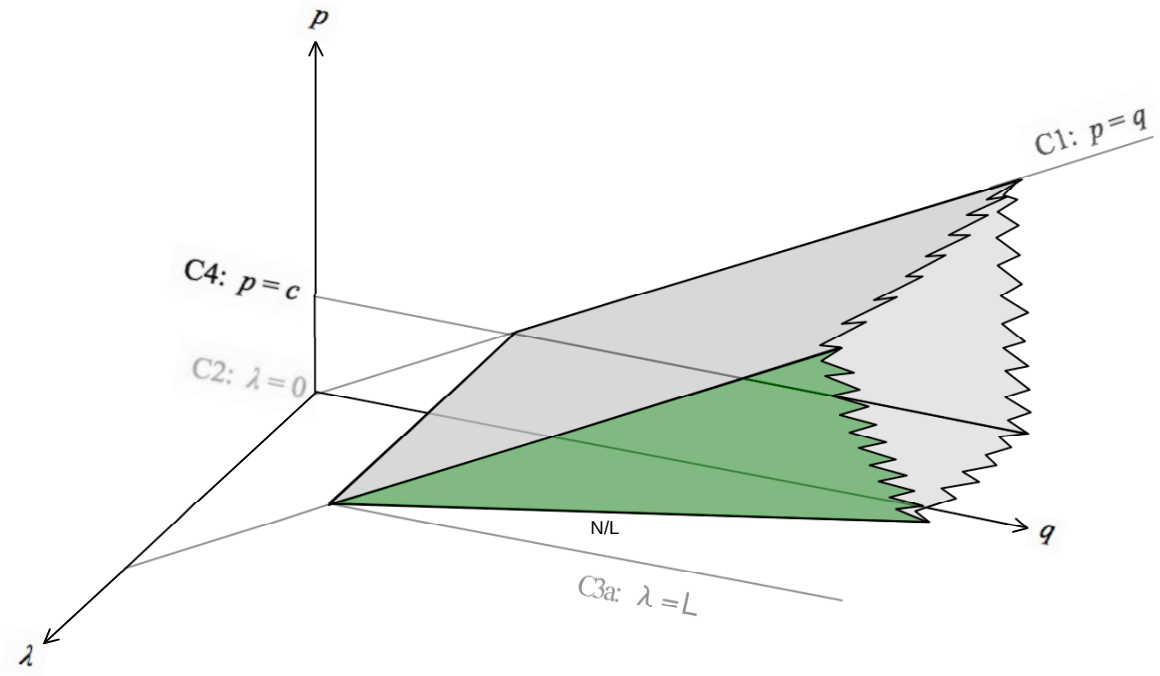
Rationally, a monopolist *not* threatened by potential entry will not finance a horizontal expansion that, in combination with the attraction effect of maximum consumer surplus ($q - p$), would lead to an *attraction range exceeding full market coverage* (the consumer spectrum L).

Please note that, other than Constraint 3a, the logic of this constraint becomes void under threat of entry, as it may well be favourable to push one's horizontal expansion out beyond the point of full market coverage in order to *outcompete* a potential entrant and/or prevent his entry and competition in the extremal regions of the consumer spectrum.

Thus, Constraint 3b can be formulated as

$$\lambda^* \leq L - 2/\tau \cdot (q - p)$$

[35] in the perfectly equivalent form of $\pi(q) = N \cdot (q - c) - k_q \cdot q^2 - k_\lambda \cdot L^2 \rightarrow \max !$



Graphics A.VII: Marginal solution 3b: Max. market coverage ($\lambda = L - 2/\tau \cdot (q - p)$)

The original profit function

$$\pi(\lambda, p, q) = N/L \ (\lambda + 2/\tau\,(q - p)) \cdot (p - c) - k_q \cdot q^2 - k_\lambda \cdot \lambda^2 \rightarrow \max !$$

is, in **Marginal solution 3b** ($\lambda = L - 2/\tau \cdot (q - p)$), reduced to

MS 3b: $$\pi(p, q) = N \cdot (p - c) - k_q \cdot q^2 - k_\lambda \cdot [\, L - 2/\tau\,(q - p)\,]^2 \rightarrow \max !$$

of which we can already see (by comparison) that it is **systematically superior to Marginal solution 3a**

MS 3a: $$\pi(p, q) = N \cdot (p - c) - k_q \cdot q^2 - k_\lambda \cdot L^2 \rightarrow \max !$$

discussed above in A.1.6, as the quadratic expansion costs term in MS 3b is necessarily smaller.

First order conditions are

$$\partial\pi/\partial p = N - 4\,k_\lambda/\tau\ [L - 2/\tau\ (q - p)] = 0$$

$$\partial\pi/\partial q = -2\,k_q\,q + 4\,k_\lambda/\tau\ [L - 2/\tau\ (q - p)] = 0$$

Using Cramer's Rule to solve the 2 x 2 linear equations system,

$$\begin{bmatrix} -8\,k_\lambda/\tau^2 & +8\,k_\lambda/\tau^2 \\ +8\,k_\lambda/\tau^2 & -2\,k_q - 8\,k_\lambda/\tau^2 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} -N + 4\,k_\lambda L/\tau \\ -4\,k_\lambda L/\tau \end{bmatrix}$$

$$\det H = \begin{vmatrix} -8\,k_\lambda/\tau^2 & +8\,k_\lambda/\tau^2 \\ +8\,k_\lambda/\tau^2 & -2\,k_q - 8\,k_\lambda/\tau^2 \end{vmatrix} = 16\,k_q\,k_\lambda/\tau^2$$

$$\det H_p = \begin{vmatrix} -N + 4\,k_\lambda L/\tau & +8\,k_\lambda/\tau^2 \\ -4\,k_\lambda L/\tau & -2\,k_q - 8\,k_\lambda/\tau^2 \end{vmatrix} = 2\,k_q\,N - 8\,k_\lambda\,k_q\,L/\tau + 8\,k_\lambda\,N/\tau^2$$

$$\det H_q = \begin{vmatrix} -8\,k_\lambda/\tau^2 & -N + 4\,k_\lambda L/\tau \\ +8\,k_\lambda/\tau^2 & -4\,k_\lambda L/\tau \end{vmatrix} = 8\,k_\lambda N/\tau^2$$

we get an inner optimum at

$$p^* = \det H_p / \det H = N/2k_q + N\,\tau^2/8k_\lambda - L\,\tau/2$$

$$q^* = \det H_q / \det H = N/2k_q > 0$$

Critical profit equals

$$\pi^*(p^*, q^*) = N \cdot (N/4k_q + N\,\tau^2/16k_\lambda - L\,\tau^2/2 - c)$$

q^* exceeds p^* as long as $(N/L < 4\,k_\lambda/\tau)$ is true, else (in case Constraint 1 ($p \leq q$) is not thus met) the solution converges towards Constraints 1+3[36] (see section A.1.4). Please note that at Constraint 1 ($p = q$), there is no longer a difference between Constraint 3a and Constraint 3b (see Graphics A.II and A.III, respectively).

The principal minors of $\pi(x, z)$ are

$$\det H_{1,1} = -8\,k_\lambda/\tau^2 < 0$$

$$\det H_{2,2} = 16\,k_q\,k_\lambda/\tau^2 > 0$$

Therefore, the critical point is necessarily a maximum.

We conclude that

- at C3b, a **maximum** necessarily exists,
- with profit equal to the constant term,
- at $\boldsymbol{p^* = N/2k_q + N\tau^2/8k_\lambda - L\tau/2}$

  $\boldsymbol{q^* = N/2k_q}$

  $\boldsymbol{\lambda^* = N\tau/4k_\lambda}$

- This solution may *still* violate the non-negative consumer surplus constraint $p^* \leq q^*$ (it does if $N/L > 4\,k_\lambda/\tau$ is true); then but only then **corner solution** from Constraints 1+3

  $\boldsymbol{p^* = q^* = N/2k_q}$

  $\boldsymbol{\lambda^* = L}$

  is optimal (see section A.1.4, *MS 1+3*).

[36] The only other possible conversion being Constraint 4 (marginal cost pricing), where a valid solution can conclusively be ruled out, as shown in the following section A.1.8.

- Either way, the **market** is **fully covered**.

- If we include the zero option as the optimal strategy in case of otherwise negative profits, the ***complete*** **strategic set** is

$$q^* = \begin{cases} N/2k_q & \text{if } \pi^* \geq 0 \\ 0 & \text{if } \pi^* < 0 \end{cases}$$

$$p^* = \begin{cases} \min\limits_{q^*} \left\{ N/2k_q + N\tau^2/8k_\lambda - L\tau/2 \right\} & \text{if } \pi^* \geq 0 \\ 0 & \text{if } \pi^* < 0 \end{cases}$$

$$\lambda^* = \begin{cases} N\tau/4k_\lambda & \text{if } \pi^* \geq 0 \\ 0 & \text{if } \pi^* < 0 \end{cases}$$

As the next section will rule out Marginal solution 4 as a valid solution, **this strategic set is conclusively shown to be the optimal strategy for a monopolist *not* under threat of entry.**

Excursus: Optimality of full market coverage *without* the possibility to expand

In view of *progressive* costs to cover market shares (*quadratic* cost terms for both quality and expansion), the optimality of ***full*** **market coverage** may come as quite a surprise. It can be shown though, that even *lacking* the advantageous possibility to attract via expansion (e.g. due to infinitely high expansion costs: $k_\lambda = \infty$), full market coverage is *still* optimal:

$\pi(\lambda, p, q) = N/L\ [\lambda + 2/\tau \cdot (q - p)] \cdot (p - c) - k_q \cdot q^2 - k_\lambda \cdot \lambda^2 \rightarrow \max !$

$\pi(p, q) = 2N/L\tau \cdot (q - p) \cdot (p - c) - k_q \cdot q^2 \rightarrow \max !$

$\partial\pi/\partial p = 2N/L\tau \cdot (q - 2p + c) = 0$

$\partial\pi/\partial q = 2N/L\tau \cdot (p - c) - 2\, k_q \cdot q = 0$

$p^* = c + c\, k_q\, \tau / (N/L - 2\, k_q\, \tau) > c$ if $N/L > 2\, k_q\, \tau$

$q^* = (c\, N/L) / (N/L - 2\, k_q\, \tau) > p^*$ if $N/L > 2\, k_q\, \tau$

$\pi^* = -c^2\, k_q\, N/L / (N/L - 2\, k_q\, \tau) > 0$ if $N/L < 2\, k_q\, \tau$

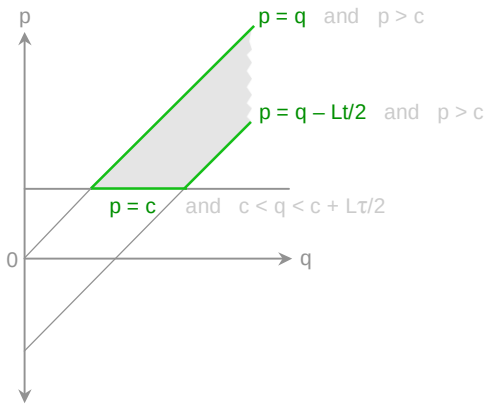
$$H = \begin{Bmatrix} -4\, N/L\tau & 2\, N/L\tau \\ 2\, N/L\tau & -2\, k_q \end{Bmatrix}$$

$\det H_{1,1} = -4\, N/L\tau < 0$

$\det H_{2,2} = 8\, k_q\, N/L\tau - 4\, N^2/L^2\tau^2 > 0$ if $N/L < 2\, k_q\, \tau$

→ no valid optimum as every profit maximum ($N/L < 2\, k_q\, \tau$) occurs outside valid parameter space ($N/L > 2\, k_q\, \tau$), i.e. where the constraints for p^* and q^* ($p^* > c$, $q^* > p^*$) are met.

Therefore, the valid solution has to be a marginal solution. Of the emerging three,



Graphics A.VIII: Marginal solutions if expansion is impossible

two can easily be ruled out:

$p = p_{max} = q \quad \rightarrow$ without consumer surplus *nor* expansion, no consumer buys

$p = p_{min} = c \quad \rightarrow$ no profit

leaving us, again, with the full market coverage solution:

$2/\tau \ (q - p) = L \quad \rightarrow$ full market coverage

$\Leftrightarrow \quad p = q - L\tau/2 \quad \Rightarrow \quad p^* < q^*$

$\pi(q) = N \cdot (q - L\tau/2 - c) - k_q \cdot q^2$

$d\pi/dq = N - 2\,k_q \cdot q = 0 \qquad d^2\pi/dq^2 = -2\,k_q < 0 \rightarrow$ maximum

$\Rightarrow \quad \boldsymbol{q^* = N/2k_q}$

$\Rightarrow \quad \boldsymbol{p^* = N/2k_q - L\tau/2}$

Clearly, this conforms with the *general* (MS3b) solution of $q^* = N/2k_q$ and $p^* = N/2k_q - L\tau/2 + N\tau^2/8k_\lambda$ whose last term disappears if $k_\lambda \rightarrow \infty$.

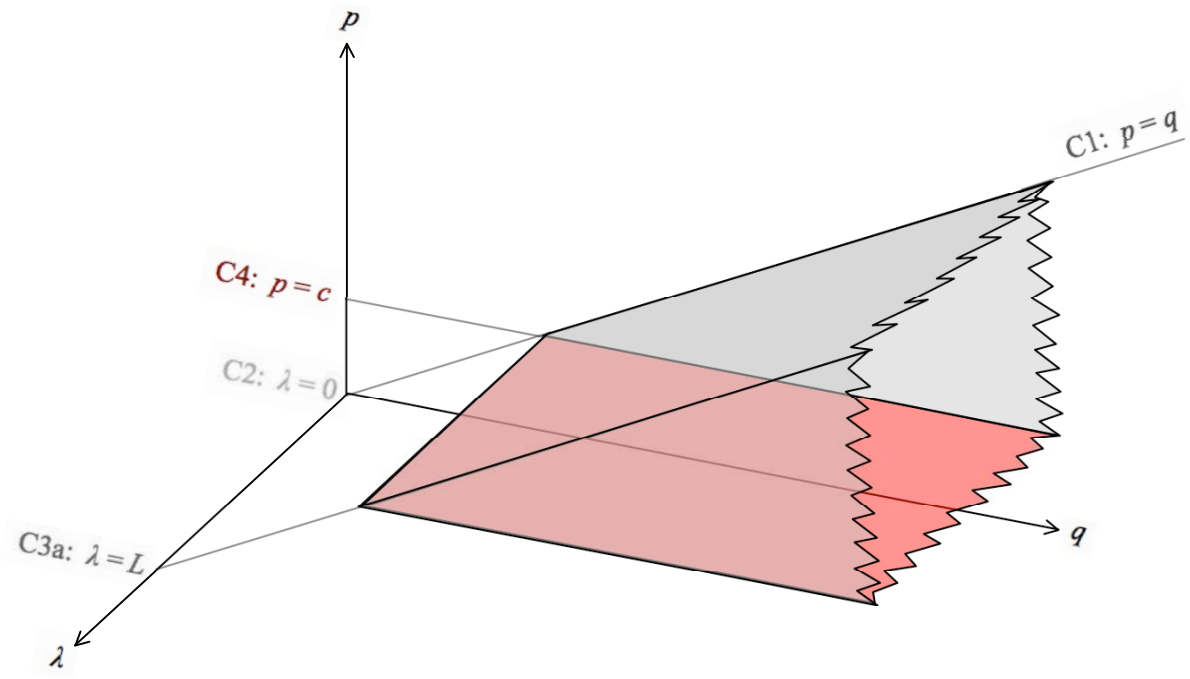
Now as full market coverage has been shown to be optimal even *without* the possibility to attract via expansion, we can a forteriori conclude that *given* the possibility to horizontally expand ($k_\lambda << \infty$), full market coverage must be optimal *all the more* – as we add the possibility of expansion in order to more easily attain full market coverage, we incrementally increase λ^*, which successively substitutes costly quality/consumer surplus while costing nothing:

$K_\lambda(\lambda \approx 0) = k_\lambda \cdot \lambda^2 \approx 0$

The above solution can only be shown to be the optimal solution if we can rule out the last remaining marginal solution (MS4) as well, which will quickly be accomplished in the following section:

A.1.8 Marginal solution 4: Minimum price ($p = c$)

Constraint 4 states that price may not undercut marginal costs ($p \geq c$), else the profit margin and therefore profits be negative. The corresponding marginal solution is, therefore, a minimum price set equal to marginal costs.



Graphics A.IX: Marginal solution 4: Minimum price ($p = c$)

With $p = c$, the original profit function

$$\pi(\lambda, p, q) = N/L \ (\lambda + 2/\tau \ (q - p)) \cdot (p - c) - k_q \cdot q^2 - k_\lambda \cdot \lambda^2 \rightarrow \max !$$

instantly collapses to a mere cost function:

$$\pi(\lambda, q) = - k_q \cdot q^2 - k_\lambda \cdot \lambda^2 \rightarrow \max !$$

where the **zero option** ($\lambda^* = 0, \ q^* = 0$) becomes the obvious optimal choice.

We can therefore conclusively rule out Marginal solution 4 as a valid solution.

A.2 Closed-form solution for Entry Deterrence

A.2.1 *Central entry and competition* calculation

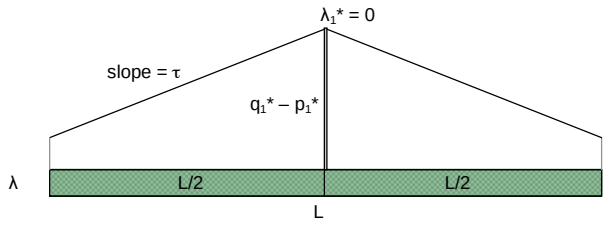
Assumption: Both the incumbent and the entrant would locate in the very centre of the consumer spectrum (at $L/2$) and compete by **maximizing** their respective (index i) **central consumer surplus** ($q_i - p_i$).

$u_{L/2}(p_i, q_i) = q_i - p_i \rightarrow \max !$

The incumbent settles in the centre for reasons of symmetry that might be quite intuitive: In case of threat of entry, the entrant is not “invited” to easily locate on either “free” side. Without threat of entry or in case of preemption, it is easiest to reach both ends of characteristic space and thus (potentially) cover the entire market (full market coverage, FMC).

Two cases need to be distinguished:

- Optimal maximum consumer surplus ($q_1^* - p_1^*$) alone is sufficient to cover the entire market L. This case is discussed below.
- Optimal maximum consumer surplus ($q_1^* - p_1^*$) is *not* sufficient to cover the entire market L.
  This case will be discussed after.



Graphics A.X: **Optimal** ($q_1^* - p_1^*$) **sufficient to cover the entire market**

Assuming the first case, it becomes clear that horizontal expansion λ is a mere cost factor (see below), as it does not lead to additional peripherally located consumers (they are, by the assumption of market coverage, already purchasing, because quality and price ($q_1^* - p_1^*$) *do* offer them a positive net surplus at every location of L). The according profit function

$$\pi_1(\lambda_1, p_1, q_1) = N\ (p_1 - c)\ - k_q \cdot q_1^2\ - k_\lambda \cdot \lambda_1^2$$

shows that, for λ , the zero option is optimal ($\lambda_1^* = 0$), leaving us with

$$\pi_1(p_1, q_1) = N\ (p_1 - c)\ - k_q \cdot q_1^2$$

Now, as any strategy the incumbent employs could be emulated (and infinitesimally undercut) by any potential entrant,[37] the incumbent is forced to annihilate his profits (as in the classical BERTRAND model). This gives us a p/q relation that annihilates profits ($p°/q°$):

$$\pi_1(p_1, q_1) = N\ (p_1 - c)\ - k_q \cdot q_1^2\ \ =!\ 0$$

$$\rightarrow\ \ p_1°(q_1) = c + k_q/N \cdot q_1^2 \quad \text{or, equivalently,} \quad q_1°(p_1) = \pm\sqrt{[N/k_q \cdot (p_1 - c)]}$$

Primordially, central consumer surplus $u_{L/2}$ needs to be maximized:

[37] subject to identical cost factors c, k_q, k_λ as the first mover, that is

$u_{L/2}(p_1, q_1) = q_1 - p_1 \rightarrow \max !$

$u_{L/2}(p_1°(q_1), q_1) = q_1 - (c + k_q/N \cdot q_1^2) \rightarrow \max !$

F.O.C: $\partial\, u_{L/2}(p_1°(q_1), q_1) / \partial q_1 = \partial/\partial q_1\ [\ q_1 - (c + k_q/N \cdot q_1^2)\ ] = 1 - 2\, k_q/N \cdot q_1 =!\ 0 \rightarrow$

$$\boldsymbol{q_1^* = N / 2k_q}$$

$$\boldsymbol{p_1^* = p_1°(q_1^*) = c + k_q/N \cdot q_1^{*2} = c + N / 4k_q}$$

$$\boldsymbol{\lambda_1^* = 0}$$

Please note that Constraint 1 ($p_1^* \leq q_1^*$) is met if $N \geq 4\, c\, k_q$, which is generally true as we assumed that optimal consumer surplus ($q_1^* - p_1^*$) was sufficient to cover the entire market L.

λ_1^* necessarily meets Constraint 2 ($\lambda \geq 0$) and Constraint 3 ($\lambda \leq L$).

p_1^* necessarily meets Constraint 4 ($p \geq c$).

S.O.C. show that this strategic configuration is indeed a **maximum**:

$$\partial^2 u_{L/2}(p_1) / \partial p_1^2 = - N^2/4k_q \cdot [N/k_q \cdot (p_1 - c)]^{-3/2} < 0 \quad \text{as} \quad p_1 > c$$

$$\partial^2 u_{L/2}(p_1) / \partial p_1\, q_1 = 0$$

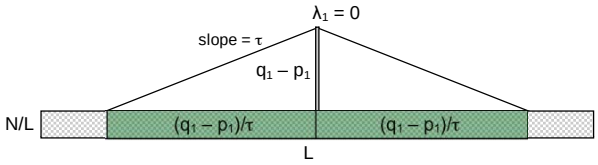
$$\partial^2 u_{L/2}(q_1) / \partial q_1^2 = - 2\, k_q/N < 0$$

$$\text{Hessian H} = \begin{bmatrix} - N^2/4k_q \cdot [N/k_q \cdot (p_1 - c)]^{-3/2} & 0 \\ 0 & - 2\, k_q/N \end{bmatrix}$$

$$\Delta_1(\text{H}) = - N^2/4k_q \cdot [N/k_q \cdot (p_1 - c)]^{-3/2} < 0 \quad \text{as} \quad p_1 > c \mid \forall\, p_1^*$$

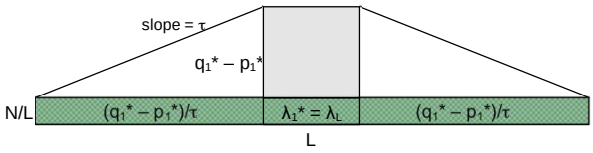
$$\Delta_2(\text{H}) = + N/2 \cdot [N/k_q \cdot (p_1 - c)]^{-3/2} > 0 \quad \text{as} \quad p_1 > c \mid \forall\, p_1^*$$

In the second case, **optimal maximum consumer surplus** ($q_1^* - p_1^*$) **is *not* sufficient to cover the market** (as shown in Graphics 4.XI):



Graphics A.XI: ($q_1^* - p_1^*$) *not* sufficient to cover the market

Analogously to the monopoly case discussed in A.1 (A.1.1 through A.1.7), the inner equilibrium and marginal solutions except for *Marginal solution 3b: Full market coverage* can be ruled out as valid or optimal solutions, leaving us, again, with full market coverage as the dominant strategy:[38] It is consumer-surplus maximizing and therefore optimal to employ a **positive horizontal expansion $\lambda_1^* > 0$ that *exactly* covers the market** (as shown below in Graphics A.XII):

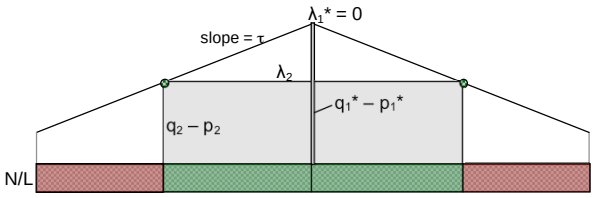


Graphics A.XII: Horizontal expansion employed to exactly cover the market ($\lambda_1^* > 0$)

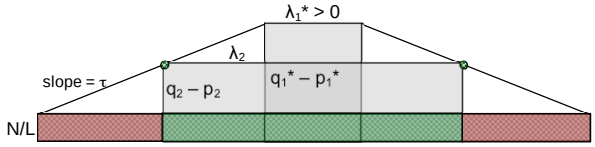
Either way, ***Central entry and competition* is not a-priori preemptive**, as an entrant may still outcompete the incumbent by carrying a lower (non-maximized) level of consumer surplus ($q_2 - p_2$) farther into the periphery (by employing a λ_2 higher than λ_1^*), thus offering a

[38] optimal *if profitable* that is, else the zero option be optimal

(marginally) higher level of net surplus to the consumers located there (reddishly shaded locations in Graphics A.XIII and A.XIV, respectively):



Graphics A.XIII: Outcompeting the incumbent peripherally ($\lambda_1^* = 0$)



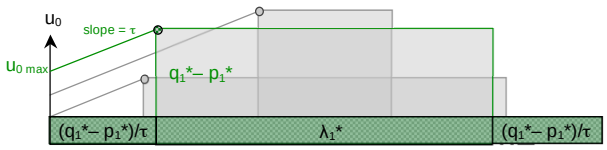
Graphics A.XIV: Outcompeting the incumbent peripherally ($\lambda_1^* > 0$)

The optimal variant of this strategy will be discussed in the following section (A.2.2 *Central entry and peripheral competition*).

A.2.2 *Central entry and peripheral competition* calculation

Again, two cases need to be distinguished:

- Optimal maximum consumer surplus ($q_1^* - p_1^*$) alone is sufficient to cover the entire market L. This case is discussed below.
- Optimal maximum consumer surplus ($q_1^* - p_1^*$) is *not* sufficient to cover the entire market L. This case will be discussed after.



Graphics A.XV: Maximizing peripheral consumer surplus u_0 in case of **full market coverage (FMC)**

The incumbent's (indexed 1) maximization of peripheral net consumer surplus u_0

$$u_0(\lambda_1, p_1, q_1) = q_1 - p_1 + \tau\,(\lambda_1/2 - L/2) \rightarrow \max!$$

still needs to be profit-maximizing, given any level of attained u_0 (otherwise the same level of u_0 could be offered more efficiently / at lower costs). As we assume full market coverage (FMC), the incumbent's profit function equals

$$\pi_{1\,FMC}(\lambda_1, p_1, q_1) = N\,(p_1 - c) - k_q \cdot q_1^2 - k_\lambda \cdot \lambda_1^2 \rightarrow \max!$$

First, we normalize marginal utilities $\partial u_0 / \partial x_1$ by the respective corresponding investment $\partial \pi / \partial x$ to obtain *marginal utility gain per profit unit invested:*

$$\partial u_0 / \partial \lambda_1 = \tau/2 \qquad \partial \pi_{1\,FMC} / \partial \lambda_1 = -2\,k_\lambda \cdot \lambda_1$$

$$\partial u_0 / \partial p_1 = -1 \qquad \partial \pi_{1\,FMC} / \partial p_1 = N$$

$$\partial u_0 / \partial q_1 = 1 \qquad \partial \pi_{1\,FMC} / \partial q_1 = -2\,k_q \cdot q_1$$

$$(\partial u_0 / \partial \lambda_1) \;/\; (\partial \pi_{1\,FMC} / \partial \lambda_1) = \tau/2 \;/\; (-2\,k_\lambda \cdot \lambda_1)$$

$$(\partial u_0 / \partial p_1) \;/\; (\partial \pi_{1\,FMC} / \partial p_1) = -1 / N$$

$$(\partial u_0 / \partial q_1) \;/\; (\partial \pi_{1\,FMC} / \partial q_1) = 1 / (-2\,k_q \cdot q_1)$$

Furthermore, *normalized marginal utilities* must be *equal*, else a mere shift in investment (a transfer of invested profit units from one cost centre to another, e.g. from quality to horizontal expansion or to a lower price) could still increase u_0.

F.O.C. 1+2:

$$(\partial u_0 / \partial \lambda_1) \;/\; (\partial \pi_{1\,\mathrm{FMC}} / \partial \lambda_1) \;=\; (\partial u_0 / \partial p_1) \;/\; (\partial \pi_{1\,\mathrm{FMC}} / \partial p_1) \;=\; (\partial u_0 / \partial q_1) \;/\; (\partial \pi_{1\,\mathrm{FMC}} / \partial q_1)$$

$$\tau/2 \;/\; (-2\, k_\lambda \cdot \lambda_1^*) \;=\; -1 / N \;=\; 1 / (-2\, k_q \cdot q_1^*)$$

From this, we already get

$$\boldsymbol{q_1^* \;=\; N \,/\, 2k_q}$$

$$\boldsymbol{\lambda_1^* \;=\; N\,\tau \,/\, 4k_\lambda}$$

Optimal price p_1^* we get from F.O.C. 3 (BERTRAND: no profit left, else the incumbent's strategy be emulated and incrementally undercut):

F.O.C. 3:

$$\pi_1(\lambda_1^*, p_1^*, q_1^*) \;=\; N\;(p_1^* - c) \;- k_q \cdot q_1^{*2} \;- k_\lambda \cdot \lambda_1^{*2} \;=\; 0$$

$$\rightarrow\; \boldsymbol{p_1^* \;=\; c \;+\; N \,/\, 4k_q \;+\; N\,\tau^2 \,/\, 16k_\lambda}$$

Thus, the stable strategic set efficiently maximizing u_0 and renouncing to profits, on condition of **full market coverage (FMC)**, is:

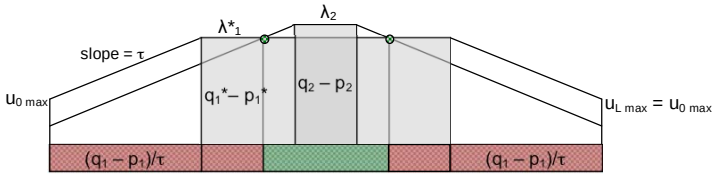
$$p^* = \left\{ \begin{array}{ll} \min \left\{ \begin{array}{c} c + N/4\,k_q + N\tau^2/16\,k_\lambda \\ q^* \end{array} \right\} & \text{if } \pi^* \geq 0 \\ 0 & \text{if } \pi^* < 0 \end{array} \right\}$$

$$q^* = \left\{ \begin{array}{ll} N/2k_q & \text{if } \pi^* \geq 0 \\ 0 & \text{if } \pi^* < 0 \end{array} \right\}$$

$$\lambda^* = \left\{ \begin{array}{ll} N\,\tau\,/\,4k_\lambda & \text{if } \pi^* \geq 0 \\ 0 & \text{if } \pi^* < 0 \end{array} \right\}$$

From a central location, this deterrence strategy (λ_1^*, p_1^*, q_1^*) cannot be outcompeted *peripherally*.

In order to prove that it is **preemptive**, we first need to show that, in addition, it is impossible for any centrally locating entrant (indexed 2) to outcompete it *centrally*, too, which could be achieved by offering a higher benefit near the centre by offering a higher maximum consumer surplus ($q_2 - p_2$) but *not* investing in carrying it out into the periphery via an increased λ_2.
Second, we'll need to show that no peripherally located entrant can profitably outcompete the incumbent in the periphery; this case will be discussed in section A.2.3 *Peripheral entry and competition.*
The above deterrence strategy is actually preemptive if it prevents *either* kind of entry.



Graphics A.XVI: Centrally outcompeting the incumbent's peripherally optimized deterrence strategy

The length of the central segment of the market that, hypothetically[39], could be captured by an entrant, equals

$\lambda_2 + 2/\tau \cdot (q_2 - p_2 - q_1^* + p_1^*)$

Resulting profit would equal

$$\pi_2(\lambda_2, p_2, q_2) = N/L\ [\ \lambda_2 + 2/\tau \cdot (q_2 - p_2 - q_1^* + p_1^*)\ ] \cdot (p_2 - c) - k_q \cdot q_2^2 - k_\lambda \cdot \lambda_2^2 \ \rightarrow \max!$$

$$= N/L\ [\ \lambda_2 + 2/\tau\ (q_2 - p_2 - N/2k_q + c + N/4k_q + N\,\tau^2/16k_\lambda)\ ] \cdot (p_2 - c) - k_q \cdot q_2^2 - k_\lambda \cdot \lambda_2^2 \ \rightarrow \max!$$

[39] if profitable

But even maximizing profit π_2 by employing the following **optimal response** strategy,

F.O.C.:

$$\partial\, \pi_2(\lambda_2, p_2, q_2) / \partial\lambda_2 \;=\; N/L \cdot (p_2 - c) - 2\, k_\lambda \cdot \lambda_2 \;=!\; 0$$

$$\partial\, \pi_2(\lambda_2, p_2, q_2) / \partial p_2 \;=\; N/L \cdot \lambda_2 + N/L \cdot (2/\tau) \cdot (q_2 - 2\, p_2 - N / 4k_q + 2\, c + N\, \tau^2 / 16k_\lambda) \;=!\; 0$$

$$\partial\, \pi_2(\lambda_2, p_2, q_2) / \partial q_2 \;=\; N/L \cdot (2/\tau) \cdot (p_2 - c) - 2\, k_q \cdot q_2 \;=!\; 0$$

$\rightarrow$

$$\boldsymbol{\lambda_2^* \;=\; (-\, k_q\, N^2\, \tau^3 + 4\, k_\lambda\, N^2\, \tau) \,/\, (8\, k_\lambda) \,/\, (k_q\, N\, \tau^2 + 4\, k_\lambda\, N - 8\, k_q\, k_\lambda\, L\, \tau)}$$

$$\boldsymbol{p_2^* \;=\; (-\, k_q\, N\, \tau^2 \cdot (4\, c - L\, \tau) + 4\, k_\lambda\, N \cdot (L\, \tau + 4\, c) - 32\, c\, k_q\, k_\lambda\, L\, \tau) \,/\, 4 \,/\, (k_q\, N\, \tau^2 + 4\, k_\lambda\, N - 8\, k_q\, k_\lambda\, L\, \tau)}$$

$$\boldsymbol{q_2^* \;=\; (-\, k_q\, N^2\, \tau^2 + 4\, k_\lambda\, N^2) \,/\, (4\, k_q) \,/\, (k_q\, N\, \tau^2 + 4\, k_\lambda\, N - 8\, k_q\, k_\lambda\, L\, \tau)}$$

$\rightarrow$

$$\pi_2^*(\lambda_2^*, p_2^*, q_2^*) \;=\; N \cdot (p_2^* - c) - k_q \cdot q_2^{*2} - k_\lambda \cdot \lambda_2^{*2}$$

for which a valid solution space

$\lambda_2^* > 0$

and

$\lambda_2^* < L$

and

$p_2^* > 0$

and

$q_2^* > 0$

and

$q_2^* - p_2^* > 0$

does exist at

$N \leq 4 k_\lambda L / \tau$

and

$k_q > (4 k_\lambda N^2 \tau - 32 k_\lambda^2 L N) / (N^2 \tau^3 + 8 k_\lambda L N \tau^2 - 64 k_\lambda^2 L^2 \tau)$

and

$c > (k_q^2 L N \tau^3 - k_q N^2 \tau^2 - 4 k_q k_\lambda L N \tau + 4 k_\lambda N^2) / (4 k_q^2 N \tau^2 - 32 k_q^2 k_\lambda L \tau + 16 k_q k_\lambda N)$

or

$4 k_\lambda L / \tau < N \leq 8 k_\lambda L \tau$

and

$k_q < 4 k_\lambda / \tau^2$

and

$c > (k_q^2 N L \tau^3 - k_q N^2 \tau^2 - 4 k_q k_\lambda L N \tau + 4 k_\lambda N^2) / (4 k_q^2 N \tau^2 - 32 k_q^2 k_\lambda L \tau + 16 k_q k_\lambda N)$

or

$N > 8 k_\lambda L \tau$

and

$4 k_\lambda / \tau^2 > k_q > (4 k_\lambda N^2 \tau - 32 k_\lambda^2 L N) / (N^2 \tau^3 + 8 k_\lambda L N \tau^2 - 64 k_\lambda^2 L^2 \tau)$

and

$c < (k_q^2 L N \tau^3 - k_q N^2 \tau^2 - 4 k_q k_\lambda L N \tau + 4 k_\lambda N^2) / (4 k_q^2 N \tau^2 - 32 k_q^2 k_\lambda L \tau + 16 k_q k_\lambda N)$

solving the inequality system shows that **no** valid **parameter configuration**

$c > 0$

$k_q > 0$

$k_\lambda > 0$

$L > 0$

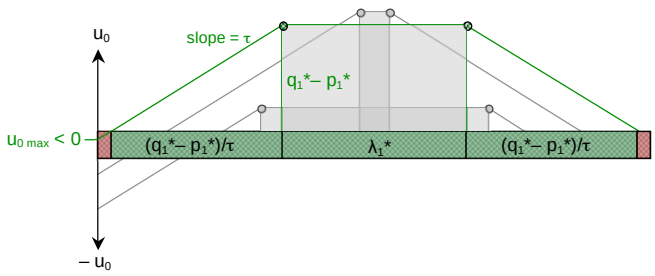
$N > 0$

$\tau > 0$

exists that would allow for non-negative profits $\pi_2 \geq 0$, given the incumbent's deterrence strategy (λ_1*, p_1*, q_1*, see above). **Therefore, *Central entry and peripheral competition* prevents** this variant of ***Central entry and competition***, even if it has been optimized.

What remains to be shown in order for a claim of complete **preemption**, is that *Central entry and peripheral competition* cannot be outcompeted by *Peripheral entry and competition*, which will be shown in section A.2.3.

The alternative case of **incomplete market coverage (IMC)** is illustrated in Graphics A.XVII:



Graphics A.XVII: Maximizing peripheral consumer surplus u_0 in case of incomplete market coverage (IMC)

Calculation proceeds analogously to the FMC case. The incumbent (indexed 1 as usual) maximizes his offered peripheral net consumer surplus u_0

$u_0(\lambda_1, p_1, q_1) = q_1 - p_1 + \tau\,(\lambda_1/2 - L/2) \rightarrow \max!$

Any level of u_0 needs to be attained while profit-maximizing (otherwise the same level of u_0 could be offered more efficiently / at lower costs). As we now assume *in*complete market coverage, the profit function remains unchanged:

$\pi_1(\lambda_1, p_1, q_1) = N/L \cdot [\,\lambda_1 + 2/\tau \cdot (q_1 - p_1)\,] \cdot (p_1 - c) - k_q \cdot q_1^2 - k_\lambda \cdot \lambda_1^2 \rightarrow \max!$

We normalize marginal utilities $\partial u_0 / \partial x$ by the respective corresponding investment $\partial \pi_1 / \partial x$ to obtain *marginal utility gain per profit unit invested:*

$\partial u_0 / \partial \lambda_1 = \tau/2$ $\qquad$ $\partial \pi_1 / \partial \lambda_1 = N/L \cdot (p - c) - 2\,k_\lambda \cdot \lambda_1$

$\partial u_0 / \partial p_1 = -1$ $\qquad$ $\partial \pi_1 / \partial p_1 = N/L \cdot [\,\lambda_1 + 2/\tau \cdot (q_1 - 2p_1 + c)\,]$

$\partial u_0 / \partial q_1 = 1$ $\qquad$ $\partial \pi_1 / \partial q_1 = N/L \cdot (2/\tau) \cdot (p - c) - 2\,k_q \cdot q_1$

$(\partial u_0 / \partial \lambda_1) \;/\; (\partial \pi_1 / \partial \lambda_1) = \tau/2 \;/\; [\,N/L \cdot (p - c) - 2\,k_\lambda \cdot \lambda_1\,]$

$(\partial u_0 / \partial p_1) \;/\; (\partial \pi_1 / \partial p_1) = -1 \;/\; \{\,N/L \cdot [\,\lambda_1 + 2/\tau \cdot (q_1 - 2p_1 + c)\,]\,\}$

$(\partial u_0 / \partial q_1) \;/\; (\partial \pi_1 / \partial q_1) = 1 \;/\; [\,N/L \cdot (2/\tau) \cdot (p - c) - 2\,k_q \cdot q_1]$

Again, *normalized marginal utilities* must be *equal*, else a mere shift in investment (a transfer of invested profit units from one cost centre to another, e.g. from quality to horizontal expansion or to a lower price) could still increase u_0.

F.O.C. 1+2: $(\partial u_0 / \partial \lambda_1) \;/\; (\partial \pi_1 / \partial \lambda_1) = (\partial u_0 / \partial p_1) \;/\; (\partial \pi_1 / \partial p_1) = (\partial u_0 / \partial q_1) \;/\; (\partial \pi_1 / \partial q_1)$

$$\tau/2 \;/\; [\,N/L \cdot (p - c) - 2\,k_\lambda \cdot \lambda_1\,] = -1 \;/\; \{\,N/L \cdot [\,\lambda_1 + 2/\tau \cdot (q_1 - 2p_1 + c)\,]\,\} = 1 \;/\; [\,N/L \cdot (2/\tau) \cdot (p - c) - 2\,k_q \cdot q_1]$$

F.O.C. 3 results from BERTRAND competition:

$$\pi_1(\lambda_1, p_1, q_1) = N/L \cdot [\ \lambda_1 + 2/\tau \cdot (q_1 - p_1)\ ] \cdot (p_1 - c) - k_q \cdot q_1^2 - k_\lambda \cdot \lambda_1^2 =! \ 0$$

Solving for λ_1, p_1, q_1 yields

$$\lambda_1^* = 0$$
$$p_1^* = 0$$
$$q_1^* = 0$$

or

$$\boldsymbol{\lambda_1^* = 4\, c\, k_q\, N\, \tau \,/\, (k_q\, N\, \tau^2 + 4\, k_\lambda\, N - 8\, k_q\, k_\lambda\, L\, \tau)}$$

$$\boldsymbol{p_1^* = 2\, c \cdot [\ 1 + 4\, k_q\, k_\lambda\, L\, \tau \,/\, (k_q\, N\, \tau^2 + 4\, k_\lambda\, N - 8\, k_q\, k_\lambda\, L\, \tau)\ ]}$$

$$\boldsymbol{q_1^* = 8\, c\, k_\lambda\, N \,/\, (k_q\, N\, \tau^2 + 4\, k_\lambda\, N - 8\, k_q\, k_\lambda\, L\, \tau)}$$

The latter strategy *should* be optimal if the resulting profits are zero but not negative,[40] BUT it shows that the **solution for incomplete market coverage (IMC)** ***systematically overcompensates*** **incomplete market coverage** by increasing investments even more, as, analytically, the assumption of IMC falsely promises even greater profits as customers from *beyond* the actual dimension of the market (*L*) would be attracted. This leads to over-investments in attraction range, although no increased income from beyond the boundaries of *L* counterbalances those increased investments (no additional customers can actually be attracted). Thus, the **IMC solution results in (more than) full market coverage (FMC)**, and the **FMC solution is dominant**, as it optimizes on the right assumption (of FMC) to begin with.

[40] else the former (the zero option) would of course be preferable

In order to prove this conclusion, valid parameter spaces must be examined systematically:

Question 1: Does a valid parameter space exist that is *not* covered by the FMC solution and where the IMC solution does not cover (or over-cover) the entire market?

no FMC if attraction range $<$ length of characteristic space

$$\lambda_{1\,\mathrm{FMC}} + 2/\tau \cdot (q_{1\,\mathrm{FMC}} - p_{1\,\mathrm{FMC}}) < L$$

IMC if $\lambda_{1\,\mathrm{IMC}} + 2/\tau \cdot (q_{1\,\mathrm{IMC}} - p_{1\,\mathrm{IMC}}) < L$

Is there a valid (non-negative) parameter space where no-FMC and IMC are true at the same time?

– Yes, No-FMC and IMC are true at the same time if

$$k_q\, N\, \tau^2 + 4\, k_\lambda\, N - 8\, k_q\, k_\lambda\, L\, \tau < 0$$

BUT this inevitably leads to invalid results:

$\lambda_{1\,\mathrm{IMC}}^* < 0$

$q_{1\,\mathrm{IMC}}^* < 0$

Question 2: Does a valid parameter space exist where FMC is true but IMC isn't?

FMC if attraction range $\geq$ length of characteristic space

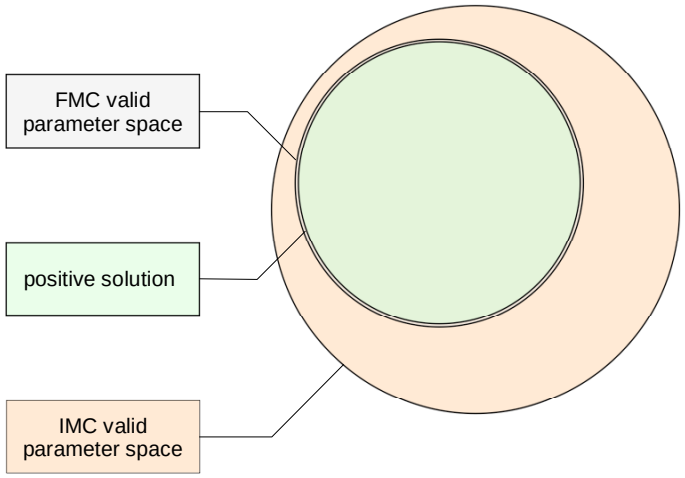
$$\lambda_{1\,\mathrm{FMC}} + 2/\tau \cdot (q_{1\,\mathrm{FMC}} - p_{1\,\mathrm{FMC}}) \geq L$$

no IMC if $\lambda_{1\,\mathrm{IMC}} + 2/\tau \cdot (q_{1\,\mathrm{IMC}} - p_{1\,\mathrm{IMC}}) \geq L$

Is there a valid (non-negative) parameter space where FMC and no-IMC are true?

– No.

From this we can conclude the following (EULER) set diagram (Graphics A.XVIII):



Graphics A.XVIII: Parameter and solution spaces of FMC and IMC

So there *is* a valid parameter space where FMC is false and IMC is true, but the resulting solution is then necessarily outside valid solution space as the strategic variables $\lambda_{1\,\mathrm{IMC}}^*$ and $q_{1\,\mathrm{IMC}}^*$ are necessarily negative.

Conclusion:

As **any solution results in full market coverage,** the **optimal FMC strategy dominates** the IMC solution, and we can rule out the IMC solution altogether.

A.2.3 *Peripheral entry and competition* calculation

In section A.2.2 we have shown that an agent settling in a *central* position (at $L/2$) and employing a full market coverage strategy maximizing peripheral consumer surplus by choosing[41]

$$p_1^* = \min \left\{ \begin{array}{c} c + N/4k_q + N\tau^2/16k_\lambda \\ q^* \end{array} \right\}$$

$$q_1^* = N/2k_q$$

$$\lambda_1^* = N\tau/4k_\lambda$$

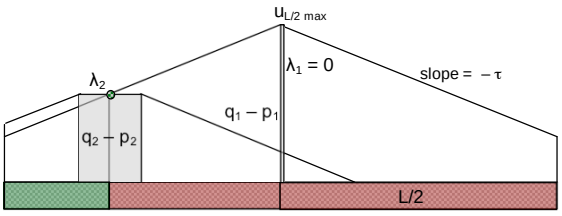
cannot be outcompeted if he/she choses to annihilate profits, but can him-/herself profitably outcompete any agent settling *centrally* not annihilating profits, be he maximizing central or peripheral consumer surplus (u_0 or $u_{L/2}$, respectively).

We will show that *Peripheral entry and competition* does not embody a viable[42] strategy to **oppose** *any* of the central strategies discussed before (***Central entry and competition*** in section A.2.1, or ***Central entry and peripheral competition*** in section A.2.2, thereby finally proving the strategic set of the latter (above p_1^*, q_1^*, λ_1^*) to be **preemptive** with respect to *all* other strategies[43]).

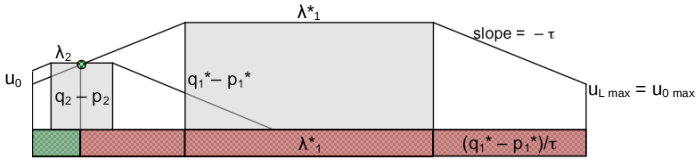
[41] provided that profits are non-negative, otherwise the zero option is of course preferable

[42] i.e., producing non-negative profits

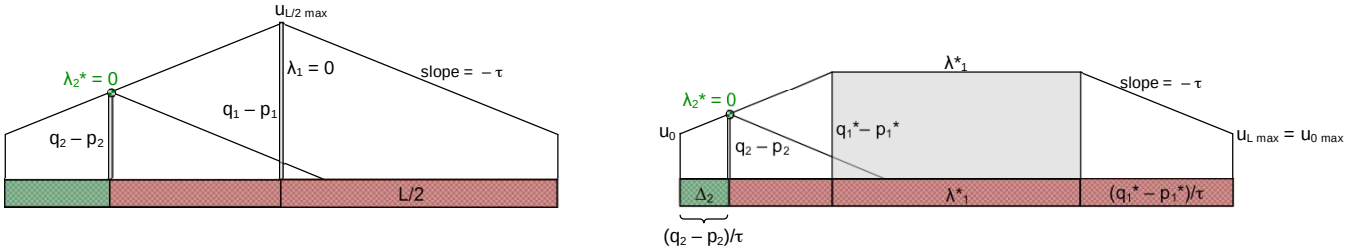
[43] as it has been shown in section A.2.2 that *Central entry and peripheral competition* outcompetes *Central entry and competition*



Graphics A.XIX: The entrant trying to outcompete the incumbent's *Central entry and competition* by locating peripherally



Graphics A.XX: The entrant trying to outcompete the incumbent's *Central entry and peripheral competition* by locating peripherally



Graphics A.XXI: Rationalizing λ_2 ($\lambda_2^* = 0$)

Comparing Graphics A.XIX, A.XX, and A.XXI, it becomes clear that investing in horizontal expansion does *not* contribute in any way to reaching any point of indifference (as usual, indicated as a green dot. Therefore, the optimal λ_2 is $\boldsymbol{\lambda_2^* = 0}$.

As a function of the entrant's location Δ_2, his profit function will equal

$$\pi_2(p_2, q_2, \Delta_2) = N/L \cdot (\Delta_2) \cdot (p_2 - c) - k_q \cdot q_2^2 \rightarrow \max!$$

In order to attain indifference at location Δ_2 (with respect to the incumbent's chosen λ_1, p_1, q_1), the entrant must offer a consumer surplus of at least [44]

$$(q_2 - p_2) \geq (q_1 - p_1) - \tau \cdot (L/2 - \Delta_2 - \lambda_1/2)$$

Rationally, he will also not offer a *higher* maximum consumer surplus than that, so we can equalize

$$(q_2 - p_2) = (q_1 - p_1) - \tau \cdot (L/2 - \Delta_2 - \lambda_1/2)$$

and substitute

$$\Delta_2 = L/2 - \lambda_1/2 + 1/\tau \cdot (q_2 - p_2 - q_1 + p_1)$$

The profit function then equals

$$\pi_2(p_2, q_2, \Delta_2) = N/L \cdot (\Delta_2) \cdot (p_2 - c) - k_q \cdot q_2^2 \rightarrow \max!$$

$$\pi_2(p_2, q_2) = N/L \cdot [L/2 - \lambda_1/2 + 1/\tau \cdot (q_2 - p_2 - q_1 + p_1)] \cdot (p_2 - c) - k_q \cdot q_2^2 \rightarrow \max!$$

[44] It's easiest to measure Δ_2 from the nearest end of the consumer spectrum L, then the formula is correct in this form, regardless of the entrant's location being left or right of the centre (Δ_2 being smaller or larger than $L/2$).

From F.O.C.

$$\partial\, \pi_2(p_2, q_2)\, /\partial p_2 \;=\; N/L\tau \cdot q_2 \;- 2\, N/L\tau \cdot p_2 \;- N/L\tau\, (q_1 - p_1) + (N/2 - N/2L \cdot \lambda_1) + cN/L\tau \;=!\; 0$$

$$\partial\, \pi_2(p_2, q_2)\, /\partial q_2 \;=\; N/L\tau \cdot p_2 \;- cN/L\tau \;- 2k_q \cdot q_2 \;=!\; 0$$

we get the **critical point** at

$$\mathbf{p_2^* \;=\; 1/(N - 4\, k_q\, L\, \tau) \cdot [\; c\, N + 2\, k_q\, L\, \tau \cdot (q_1 + \tau/2 \cdot \lambda_1 - p_1 - L\, \tau/2 - c)\;]}$$

$$\mathbf{q_2^* \;=\; N/(N - 4\, k_q\, L\, \tau) \cdot (c - L\, \tau/2 + q_1 + \tau/2 \cdot \lambda_1 - p_1)}$$

and (see above)

$$\mathbf{\lambda_2^* \;=\; 0}$$

S.O.C.

$$\partial^2\, \pi_2(p_2, q_2)\, /\, \partial p_2^2 \;\;=\; -\,2\, N/L\tau \;\; < 0$$

$$\partial^2\, \pi_2(p_2, q_2)\, /\, \partial p_2\, q_2 \;=\;\; N/L\tau \;\;\; > 0$$

$$\partial^2\, \pi_2(p_2, q_2)\, /\, \partial q_2^2 \;\;=\; -\,2\, k_q \;\;\; < 0$$

or the corresponding Hessian

$$\mathrm{H}(\pi_2) \;=\; \begin{bmatrix} -\,2\, N/L\tau & N/L\tau \\ N/L\tau & -\,2\, k_q \end{bmatrix}$$

$$\Delta_1(\mathrm{H}) \;=\; -\,2\, N/L\tau \;\; < 0$$

$$\Delta_2(\mathrm{H}) \;=\; 4\, k_q\, N/L\tau - N^2/L^2\tau^2 \;\; > 0 \quad \text{if} \quad 4\, k_q\, L\, \tau - N \;>\; 0$$

indicate the likelihood of a maximum.

But even maximizing his profits ($\pi_2^* = \pi_2(\lambda_2^*, p_2^*, q_2^*)$) by employing the optimal answer to the incumbent's ($\lambda_1^*, p_1^*, q_1^*$), solving the inequality system for non-negative profit shows that **no** valid **parameter configuration**

$c > 0$

$k_q > 0$

$k_\lambda > 0$

$L > 0$

$N > 0$

$\tau > 0$

exists that would result in a valid solution[45] generating **non-negative profits** ($\pi_2 \geq 0$), given the incumbent's strategy ($\lambda_1 = \lambda_1^*, \ p_1 = p_1^*, \ q_1 = q_1^*$); this is true for both the *Central entry and competition* optimum strategy, and the *Central entry and peripheral competition* optimum strategy.

Therefore, as neither a peripherally nor a centrally settling entrant can generate non-negative profits, be he competing in the centre or peripherally, we can conclude that the **INCUMBENT**'s deterrence strategy (above $\lambda_1^*, \ p_1^*, \ q_1^*$) **MAXIMIZING PERIPHERAL CONSUMER SURPLUS** (the *Central entry and peripheral competition* optimum) indeed takes **PREEMPTIVE** effect (effectively deters entry).

[45] $0 \leq \lambda_2^* \leq L$ (or, in the present case of peripheral settling, $L/2$)
$c \leq p_2^* \leq q_2^*$
$q_2^* > 0$

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C.2 Further reading

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Abstract

This paper will attempt to create a theoretical model integrating horizontal and vertical product differentiation as well as optimal price setting.

Horizontal differentiation will occur in two different ways: First — analogously to the standard HOTELLING model — by choosing a **location** along a spectrum of individual customer preferences (product or characteristic space), and second, by choosing a length of **horizontal expansion** extending symmetrically from the chosen location in its centre, allowing the producer to design a product simultaneously and equally covering more than one individual preference at a time.

Vertical differentiation will occur by choosing a level of produced quality equally perceived by all potential customers, regardless of the horizontal location of their individually preferred characteristic. Only one single level of quality is chosen, resulting in quasi-fixed cost independent of the number of units produced.

Optimal price setting will occur by determining one single optimal price valid for every customer regardless of their horizontal location of their preferred characteristic (*mill pricing*). I.e., a firm may *not* differentiate prices, setting different prices for different market segments/ consumers.

First, a **monopolist**'s optimal choice of the strategic variables location, horizontal expansion, quality, and price will be determined, assuming that there is **no threat of entry**.

Second, we will examine an incumbent's possible **entry deterrence** strategies in **oligopolistic competition with free entry**. On the part of the entrant, three different **strategies of entry** will be discussed: *Central entry and competition*, *Central entry and peripheral competition*, and *Peripheral entry and competition*. These must in turn be anticipated by the incumbent, whose respective deterrence strategies will be compared with regard to their **preemptive** effect.

ABOUT THE AUTHOR

Patrick Puls, born 1977 in the Federal Republic of Germany, holds an academic degree in Civil Engineering from the University of Life Sciences in Vienna, plus, by dint of this paper, an academic degree in International Business Administration from the University of Vienna.

Abstract in German / Deutschsprachige Kurzbeschreibung des Inhalts

Die vorliegende Arbeit entwickelt ein industrieökonomisches Modell, das horizontale und vertikale Produktdifferenzierung in sich vereint, und gleichzeitig auch den Preis als strategische Entscheidungsvariable behandelt.

Zur **horizontalen Produktdifferenzierung** stehen zwei Möglichkeiten zur Verfügung: Zum einen – analog zum Modell von HOTELLING (1929) – durch die Wahl eines „**Ortes**" innerhalb eines Spektrums subjektiver Präferenzen der KonsumentInnen, zum anderen, symmetrisch von diesem Ort ausgehend, durch **horizontale Expansion** in diesen Markt (Präferenzen-Raum). Dies bildet die Möglichkeit ab, Produkte anzubieten, die mehr als nur eine einzige Präferenz abdecken, sondern eine ganze *Spanne* an Präferenzen gleichermaßen nutzenbringend bedienen.

Vertikale Produktdifferenzierung geschieht durch die strategische Wahl der **Qualität**. Im Gegensatz zu und unabhängig von obigen subjektiven Präferenzen, wird Qualität von *allen* potenziellen KonsumentInnen gleichermaßen wertgeschätzt. Der Produzent kann sich nur für ein einziges Qualitätsmaß entscheiden und dieses allen KonsumentInnen anbieten. Die Kosten der produzierten Qualität sind mengenunabhängig, also quasi-fix.

Die **Preisoptimierung** erfolgt durch die Wahl eines einzigen Preises, zu dem das Produkt allen KonsumentInnen gleichermaßen angeboten wird (einheitlicher Fixpreis) – Preisdifferenzierung im Sinne unterschiedlicher Preise für verschiedene Käufersegmente ist also ausgeschlossen.

Zunächst wird der klassische **Monopol**-Fall betrachtet; hier wählt der Monopolist seine strategischen Variablen ohne durch etwaigen Markteintritt anderer Produzenten bedroht zu sein.

Danach wird die Möglichkeit des Markteintritts eines oder mehrerer Konkurrenten angenommen (also **oligolistischer Wettbewerb** bei **freiem Markteintritt**), und (ausschließlich) die entsprechenden **Abwehrstrategien** des etablierten Produzenten betrachtet. Konkurrenten können mittels dreier verschiedener Strategien auf den Markt drängen, die *zentraler Markteintritt und zentraler Wettbewerb*, *zentraler Markteintritt und Wettbewerb in der Peripherie* und *Markteintritt und Wettbewerb in der Peripherie* genannt werden. Diese drei Möglichkeiten müssen vom etablierten Produzenten antizipiert werden; seine entsprechenden Abwehrstrategien werden auf ihren **abschreckenden Effekt** hin untersucht.

KURZBIOGRAFIE

Patrick Puls, geboren 1977 in der BRD, hat einen Abschluß in Kulturtechnik und Wasserwirtschaft der Universität für Bodenkultur Wien, plus, kraft der vorliegenden Arbeit, einen Abschluß in Internationaler Betriebswirtschaft der Universität Wien.